ICL

PROP User's Guide

(Profit Rating of Projects)

1900 Series

ICL

PROP User's Guide

(Profit Rating of Projects)

1900 Series

With effect from July 1968 the name of International Computers and Tabulators Limited has been changed to International Computers Limited

Technical Publication 3389

First Edition November 1966

Reprinted November 1967
(incorporating Amendment List 1)

Issued by Technical Publications Service
International Computers Limited
Head Office: ICL House, Putney, London SW15
and printed in Great Britain by
ICL Printing Services, Letchworth, Hertfordshire

Contents

3389 (11.66)

1 Investment appraisal

INTRODUCTION 1.1

The evaluation of new investment opportunities is taking an ever increasing proportion of management's time. Yet businessmen find it difficult to delegate this responsibility because experience shows that even the most successful firms are vulnerable in this field. Investment Appraisal is the key to sustained profitability in the face of competition and recognition of this fact has brought a search for new techniques.

With the rapid pace of technological innovation, rising competition, and pressure on margins, firms can no longer afford to rely entirely on intuitive 'hunch'. This is partly because few managers have enough experience in all relevant fields to evaluate every aspect of a proposal single-handed, and partly because a firm has to put more of its capital at risk than formerly and so can no longer accept a rate of failure of projects that was once taken for granted.

In recognition of this problem, much attention has been given in recent years to developing new techniques of appraisal. These have been successfully incorporated into the decision-making processes of many large and successful firms. But many of these techniques, though not conceptually difficult to understand, require repeated and laborious calculations, and this has proved a considerable handicap to their wider acceptance and use. Their mathematical base however makes them very suitable for handling by computer.

The I.C.T. 1900 series PROP (Profit Rating of Projects) package has been designed to make available to management the most up-to-date techniques, as well as standard ones, by means of a general application computer program.

OUTLINE OF PROBLEM 1.2

Investment Appraisal can be broken down schematically into five basic operations.

1. Calculation of the net changes expected in the various cash flows of the firm owing to the implementation of the proposed project. The net cash flow of the project will be the sum of the net changes in capital investment, working capital, tax grants and allowances, marketing costs, manufacturing costs, stockholding costs, revenue from sales, corporation tax payable on profits, and any other aspect of the firm's operations thought significant by management. Incidental effects such as increased sales of other products of the firm expected to result from the project should also be included.
2. Reduction of the net cash flow to a single index of profitability against which all other projects under consideration, irrespective of their duration, nature, or cash flow pattern, can be compared.
3. Evaluation of the uncertainty inherent in the forecasts on which the cash flow estimates have been based. From this, the probability of various rates of return can be calculated.
4. Evaluation of the remaining factors in the decision (which may prove difficult to quantify in numeric terms) e.g. welfare, prestige, or the possibility of further technological innovation.
5. Rating of projects according to their overall contribution to the firm's objectives and final selection.

INVESTMENT APPRAISAL BY COMPUTER 1.3

The system adopted for the I.C.T. 1900 series PROP (Profit Rating of Projects) package is as follows.

Cash flow data is fed into the computer in batches. Each batch contains forecast cash flows for all relevant aspects of a given project under given assumptions. Computer runs are then made over selections from this data.

Each run calculates the following if required:

1 Investment grants, initial and annual tax allowances.

2 Profit, from revenue cost and income figures.

3 Corporation tax payable on profit and scrap values.

4 Net cash flows, by aggregating data for the project, allowing for lags in receipt of cash grants and in offsetting tax allowances against corporation tax liability.

5 Payback Period.

6 Average Annual Rate of Return.

7 Single-rate Discounted Cash Flow Yield solution rate of return.

8 Single-rate Discounted Cash Flow Net Present Value.

9 Dual-rate Discounted Cash Flow Yield solution rate of return.

10 Dual-rate Discounted Cash Flow Net Present Value.

11 Probability in the form of frequency distributions of different profit rates arising from uncertainty in the cash flow forecasts.

Program output for each run consists of a two-page printout giving the above information and also present and cumulative present value tables for the project at the Yield rate and at the discount rates required. Any number of runs can be performed on each batch to calculate the effects on profitability of variations in the cash flow forecasts, and any number of batches may be input.

If the probability simulation option is required, the cash flow forecasts in the batch must be sorted into groups, one group for each category of cash flow in the project. An estimate of probability is required for each forecast, so that total probability for all forecasts in any one group amounts to 100, i.e. 100%. Then a random number simulation exercise is carried out on the batch. Each run takes one selection at random from each group and calculates Discounted Cash Flow Yield, Net Present Value, or Average Annual Rate of Return for the resulting net cash flow. When the selected number of runs is completed, a printout is given showing the frequency distribution of profit that might be expected from the project under the assumptions given for that batch.

The package is designed for a wide range of applications, from a basic calculation of discounted (or compound) cash values to a detailed evaluation of profit probability for various projects in different situations. By relieving the manager of much tedious calculation the package will enable him to concentrate on improving the firm's performance in other fields, improving forecast accuracy and maximizing profits from the projects selected.

3389 (11.66)

2 Financial evaluation

There are a number of different methods of evaluating the profitability of a given cash flow. Each has its particular advantage, though one important criterion - ease of calculation by hand - is less significant when the operation is computerized. In practice, there are many variations in calculating Payback Period and Average Annual Rate of Return, mostly resulting from different ways of calculating the net cash flow, ie. with or without cash grants and tax allowances, before or after taxation on profits, and before or after internal depreciation (for replacement purposes).

Users of the I.C.T. program can omit the calculation of investment grants, tax allowances and corporation tax if they wish. But since 1) cash grants and tax allowances often represent a substantial recovery of capital, and 2) corporation tax, if not a cost, is at least a predictable levy on profit, the result of ignoring them may be a misleading profit rating.

Although 'internal depreciation for replacement purposes' may have priority as a claim on the eventual net cash flow before distribution profits are calculated, the result of deducting it from the net cash flow forecast before expected profitability is calculated would be to underestimate the rate of profit. This is because the Discounted Cash Flow (D.C.F.) Yield rate of return is calculated as the maximum rate of interest consistent with the repayment of capital plus accrued interest as soon as the pattern of cash flows from the project permits. Since this is handled 'automatically' by the calculation, any prior deduction from the net cash flow forecast of an allowance for replacement would not be consistent with the interpretation of D.C.F. Yield as defined above and its effect would be to reduce the quoted D.C.F. rate.

PAYBACK PERIOD 2.1

The Payback Period is the number of years before the capital invested (ignoring accrued interest) can be recovered from the cash flows. It is popular both for its simplicity and its conservatism. The latter however can be a severe limitation since no allowance is made for considering profits outside the payback period nor is allowance made for the greater value of money in the early years of the Payback Period, as opposed to the later. This can be seen in Figure 1.

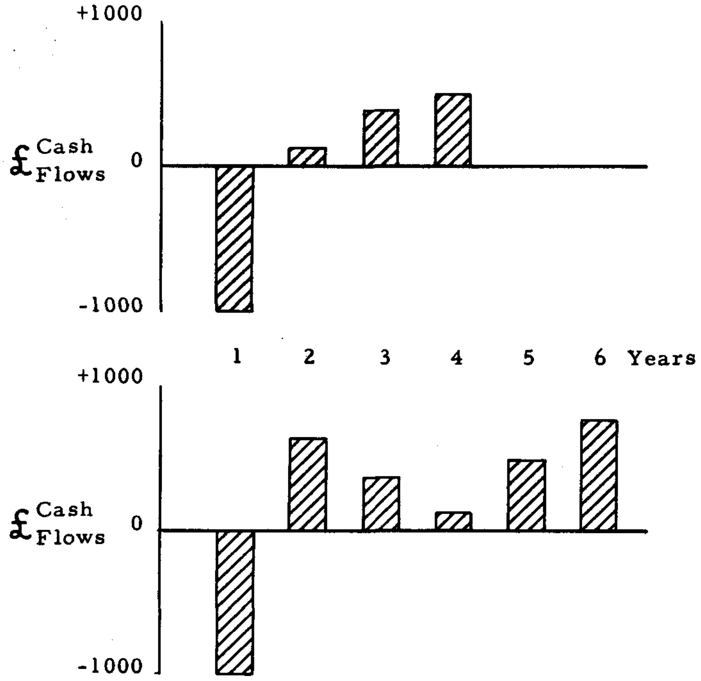
The I.C.T. program calculates Payback Period as the number of periods before the cumulative undiscounted net cash flows become zero or positive. It is useful in indicating the period during which initial capital investment is at risk This consideration is important where long-term forecasts are particularly uncertain.

AVERAGE ANNUAL RATE OF RETURN 2.2

This method is used in a variety of ways. The rate calculated in this program is equal to

$$\frac{\text{Total Net Cash Flow}}{\text{Total Capital Investment}} \times \frac{100}{\text{Life of Project in Years}}$$

This method has the advantage over Payback Period that it considers profitability over the whole life of the project, but it shares the drawback that it makes no allowance for the fact that £1 received in the first year of the project is worth more than £1 in its last year. This is because £1 received in the first year can then be reinvested to earn much more than a 'zero' rate of return for the remaining years of the project. To illustrate from Figure 1, the Average Annual Rate of Return would be the same for both projects over the first four years, yet with Project B the bulk of the capital would be repaid much sooner than with Project A, i.e. in Project B there would be much less capital at risk after year 2 than in Project A.



Project A

Payback Period three years.

Project B

Payback Period also three years yet

1 the bulk of the initial investment is recovered much earlier in the payback period and

2 over its full life of six years it is more profitable than Project A.

Figure 1 Project comparison by Payback Period

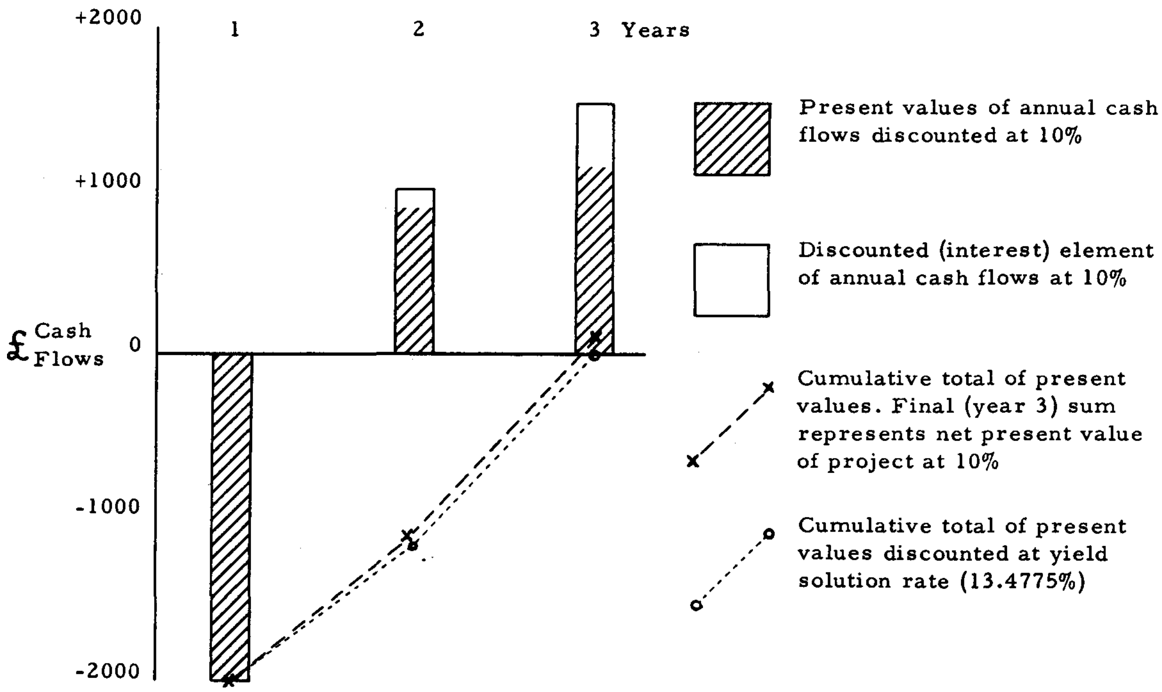


Figure 2 Single-rate D.C.F. calculation

3389 (11.66)

The particular value of these techniques is that they relate the value of money received to the time elapsed before its recovery. Their basis is simply the calculation of compound interest in reverse, i.e. backwards in time rather than forwards. For example suppose that someone invests £20 at 10% compound. In one year's time it will be worth £22, i.e. £20 × 1.1, and in two years time it will be worth £24.2, i.e. £22 × 1.1 or alternatively £20 × 1.1^2. Then looking at the situation from a different angle it could be said that £24.2 in two years time is equivalent to £20 now to a person who can earn 10% on his money. In other words the present value of £24.2 in two years' time discounted at 10% is £20, i.e. £24.2 ÷ 1.1^2. In this way the cash flows for a series of years can each be reduced to an equivalent 'base year' value for a given percentage discount, and the sum of these 'base year' or 'present' values represents the Net Present Value of the project.

Net Present Value method

For this method a discount rate is selected, usually the cost of the capital invested (if the capital is borrowed) or the opportunity cost (the rate of interest it would earn in the most profitable alternative application). Then each year's cash flow is discounted at the selected rate to calculate its present value. The sum total of all the years' present values gives the Net Present Value of the project.

A simple example (Figure 2) will illustrate the technique. Suppose Firm A invests £2000 in year 1 for an income of £990 in year 2 and £1452 in year 3. The cost to Firm A of the borrowed £2000 is 10% per annum repayable at will. The problem is to find the Net Present Value of the project at 10%. The £2000 is invested at the end of year 1, taken to be the present, therefore does not need to be discounted further since no time has elapsed between the present and the moment of cash flow. The income of £990 at end year 2 can be used to repay part of the loan and the accrued interest. By dividing £990 by 1.1 it is calculated that £900 of the principal can be repaid, also the interest for one year (£90) on the amount repaid. The present value of £990 in year 2 is thus £900, discounted at 10%, and the balance of the investment still outstanding at the end of year 2 is now -£1100. The income in year 3 is £1452, which represents a present value, discounted back over two years at 10%, of £1452 ÷ $(1.1)^2$ = £1200. The Net Present Value of the project is the sum of the present values of the cash flows, i.e. -£2000 + £900 + £1200 = £100.

One could interpret this as follows. The project requires an initial investment of £2000, which can be borrowed on the following terms - repayable at will, interest 10% compound. The cash flow at the end of year 2 could be used to repay £900 of the borrowed capital plus one year's interest (£90). The cash flow at the end of year 3 could be used to repay the balance of the debt outstanding (£1100) plus two years' accrued interest (£231). The remainder of the cash flow in year 3 (£121) represents the profit on the project which, allowing for the intervening two years before it is received, is equivalent to a present value of £100. Note that the profit is discounted back to present value at the same rate as the interest on the capital repayments. This implies that the marginal rate of interest that the firm can earn on funds at its disposal is the same as the rate the firm pays on capital borrowed for the project. This assumption is unavoidable if a single-rate D.C.F. method is chosen. The dual-rate method offers more flexibility in this respect (see subsection 2.4).

The Net Present Value method is useful in that it does not require cash flows of mixed sign (i.e. negative and positive) as the Yield method does (see below). This means that it can be used as a method of ranking projects by cost burden where income figures are not available, as well as by profit surplus. Another advantage is that it requires less calculation than the Yield method.

Yield method

This method involves repeating the Net Present Value calculation with different values of the discount rate until the rate is found at which the Net Present Value of the project approximates to zero. In other words the Yield solution rate of return is that rate at which the sum of the discounted positive cash flows equals the sum of the discounted negative cash flows. For the project outlined above in the Net Present Value example the solution yield rate is 13.4775% as follows:

			Present value
Year 1	-£2000	=	-£2000
Year 2	£990 ÷ 1.134775	=	£872.42
Year 3	£1452 ÷ $(1.134775)^2$	=	£1127.58
	Net Present Value	=	£0.0

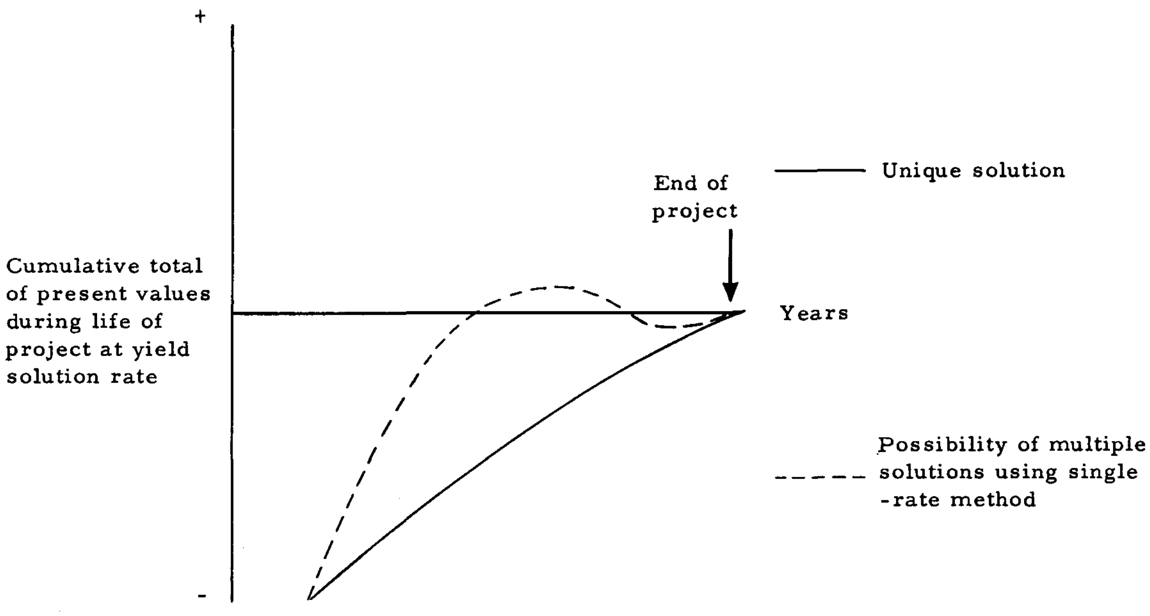


Figure 3 Occurrence of Multiple Yield solutions

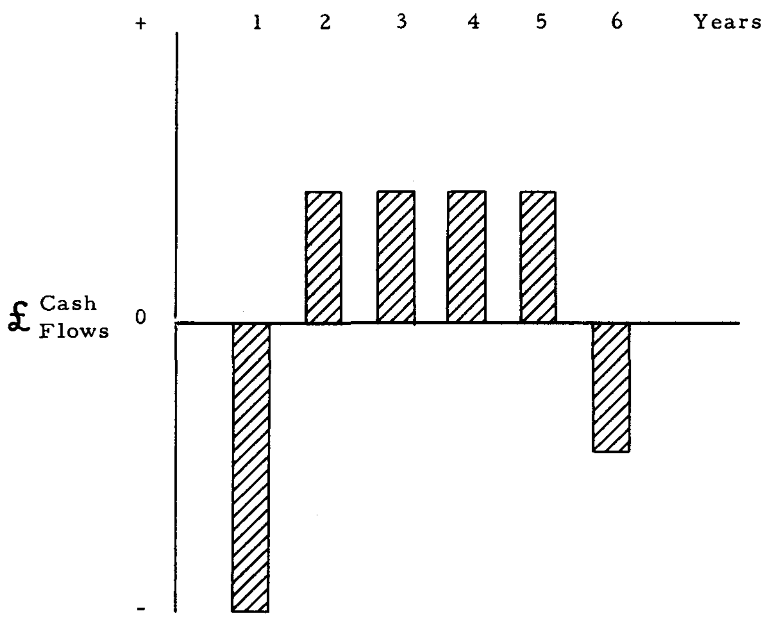


Figure 4 Multiple solution possible at single rate

3389 (11.66)

The main advantage of the Yield method is that it relates profit directly to the amount invested, i.e. it is more generally indicative of profitability to speak of a return on capital of 13.5% than a net present value of £100 at 10%. It also usefully indicates the cost ceiling for capital beyond which the project will be unprofitable. The main drawback to the Yield method is that under certain conditions multiple solutions are possible. In order to overcome this the following generalized D.C.F. method has been adopted.

DUAL-RATE DISCOUNTED CASH FLOW TECHNIQUE 2.4

A dual-rate D.C.F. technique has been adopted because it resolves two main limitations of the single -rate technique. Firstly, it solves the problem of multiple solution rates for certain cash flow patterns, and secondly it avoids using the interest rate that is paid on initial capital to discount any surplus cash flows yielded by the project after that capital is repaid.

Multiple-rate solutions to the single-rate D.C.F. Yield method may not occur often in practice but the possibility causes much uncertainty. They occur only where the cumulative total of present values changes its sign during the life of the project. But such conditions do not always produce multiple solutions. Figure 3 shows different patterns of cash flow and the possibility of multiple solutions.

A typical instance of a cash flow pattern likely to result in a multiple solution situation can be seen in Figure 4. It could be found in open-cast mining projects where land has to be restored to agricultural use after operations, thus incurring heavy negative cash flows during or at the end of the project. Similar patterns also occur where payment of corporation tax on scrap values results in a negative final cash flow.

Where a project has this pattern of cash flows the initial investment plus interest is in effect repaid at some point before the end of the project. The project then becomes a net lender so far as the firm is concerned, supplying funds which are accumulated until offset by negative cash flows later.

In many cases it is possible to resolve this difficulty by re-examining the component cash flows, since it may well indicate that two overlapping projects are being considered as one. But for cases where this cash flow pattern is unavoidable it can be resolved by discounting the cash flows when the project is a net creditor at a different rate than when it is a net debtor. This can be justified as follows. The discount rate is usually considered as the cost of capital or opportunity cost (Net Present Value method) or as the cost of capital ceiling beyond which the project is unprofitable (Yield method). This interpretation of the discount rate is not relevant when there is no longer any capital tied up in the project. During the periods when the project is a net lender of capital to the firm (either because all negative cash flows have been 'paid off', or because a sinking fund is in effect being built up to offset later negative cash flows) it is more relevant to discount cash flows at the rate that the project (and hence the firm) can earn on its surplus funds.

The basis of the dual-rate Discounted Cash Flow technique is that as long as the cumulative total of present values is negative (i.e. net outflow of funds from firm to project), the project is still paying back borrowed capital; therefore the cash flows are discounted at the cost of the capital to the project (and thus the firm). If however the cumulative total of present values becomes positive, the project has a net surplus of funds, and for as long as it stays positive cash flows are discounted at the rate earned by the project (firm) on surplus funds. This earning rate may be the marginal rate earned by the firm on its capital or an average figure.

Figure 5 gives a visual breakdown of the following dual-rate D.C.F. calculation.

	Year 1	Year 2	Year 3	Year 4
Cash Flows	-£500	+£440	+£341.5	-£231.525

Earning rate on project's surplus funds is 5%

Yield solution paying rate found to be 10%.

$$\text{Net Present Value} = £\left(-500 + \frac{440}{1.1} + \frac{341.5}{(1.1)^2}\right)\frac{(1.1)^2}{(1.05)^2} - \frac{231.525}{(1.05)^3}$$

$$= £\left(-500 + 400 + \frac{121}{(1.1)^2} + \frac{220.5}{(1.1)^2}\right)\frac{(1.1)^2}{(1.05)^2} - 200$$

$$= £0$$

This equation can be interpreted as follows:

Year 1 £500 is borrowed at a ceiling (i.e. break-even) rate of 10%.

Year 2 £400 of the debt is repaid together with accrued interest for one year of £40.

Year 3 Of a cash flow of +£341.5, £121 is used to repay the balance of debt (£100) plus accrued interest at 10% for two years £21). The remaining £220.5 is surplus income, which can be invested at 5% to amount to £231.525 by the end of year 4.

Year 4 A negative cash flow of £231.525 is offset by the accrued surplus from year 3.

In effect, a sinking fund is set up at the end of year 3 to meet negative flows later in the project, and any final Net Present Value for the project is shown discounted back to base year 1 at the earning rate if positive, or the paying rate if negative.

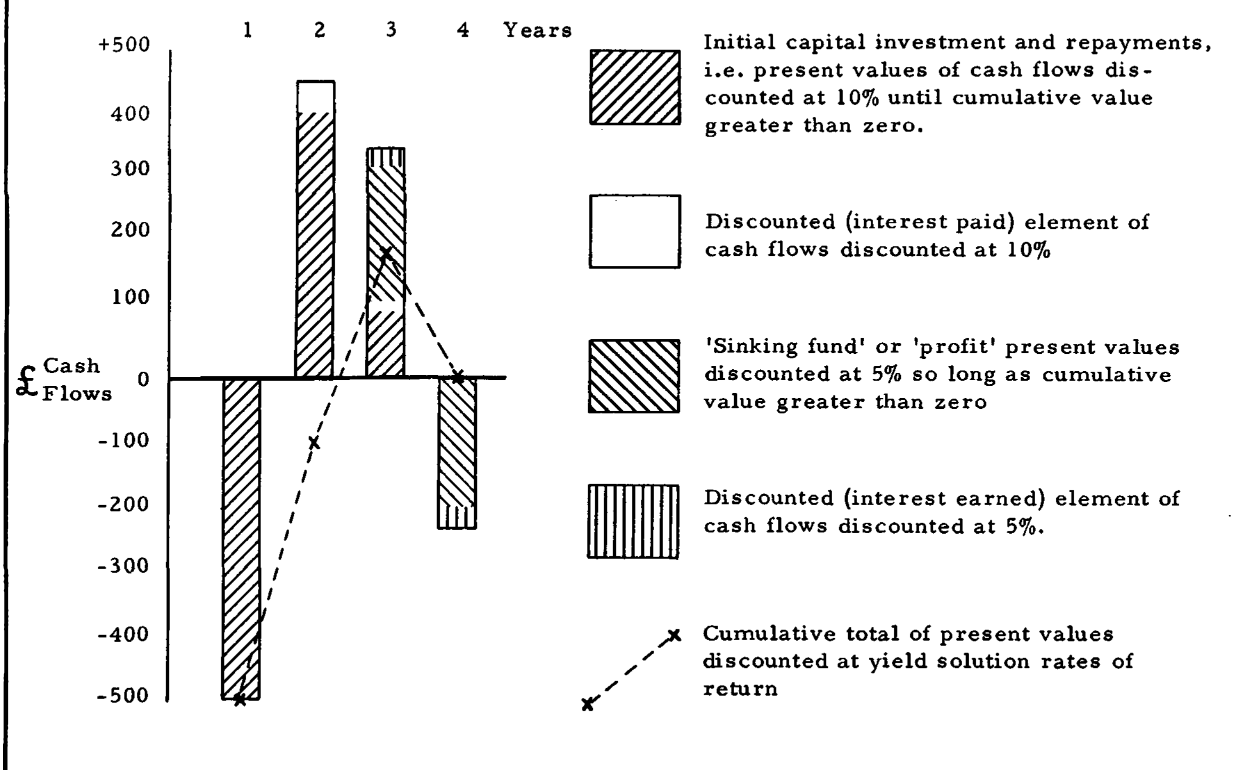


Figure 5 Dual-rate D.C.F. calculation

3389 (11.66)

Notes

1 The dual-rate D.C.F. calculation may not give the same answer as the single-rate D.C.F. method. But, in respect of the difference between the discounting of debit and credit phases of a project, it allows for a more realistic answer and prevents invalid multi-solution results.

2 In certain cases capital may not be repayable at will. The following example will illustrate this.

	Year 1	Year 2	Year 3	Year 4
Cash Flows (£)	-1000	+550	+484	+133.1

Solution Yield rate of return for this project is 10%, assuming capital repayable at will, as follows:

	Year 1	Year 2	Year 3	Year 4	Total
Present Values	-1000	+500	+400	+100	0
Interest	0	+50	+84	+33.1	167.1
Balance of Principal	-1000	-500	-100	0	

But if capital and interest are repayable only at the end of year 4, the £1000 principal will have accrued interest compounded at 10% to the sum of £331. Even if annual payment of interest is considered, this will amount to £300 over three years. In this case the solution Yield rate of 10% can be maintained only if positive cash flows can be re-invested in a sinking fund at 10% as follows:

			Total
Year 1	-1000 at 10% compound for 3 years	=	-1331.0
Year 2	+550 $\times(1.1)^2$	=	+665.5
Year 3	484×1.1	=	+532.4
Year 4	133.1	=	+133.1
			0

This assumption of re-investment of cash flows at solution Yield rate is not necessary where capital is repayable at will.

In the above case the dual-rate D.C.F. method will give the answer after some rearrangement of data. Instead of a negative cash flow of -£1000 in year 1 repayable at will, subtract from the cash flow for year 4 the £1331, which must be repaid in that year. The cash flow fed into the program will now be

Year 1	Year 2	Year 3	Year 4
0	+550	+484	-1197.9

Using the dual-rate D.C.F. method, with paying rate set at zero (since it is already included in the data input), the Yield solution earning rate will be the rate the firm must earn on money accumulating during the project in order to break even. In this case, as already seen above, it will be 10%.

Suppose that for the above example, with a 'repayable at will' Yield solution rate of 10%, the capital can be obtained only on a fixed-term three-year loan at 9%. £1000 for three years at 9% accrues to a total repayment of £1295.03 in year 4. The cash flows will now be

Year 1	Year 2	Year 3	Year 4
0	+550	+484	-1161.93

At a paying rate of zero, the minimum rate the firm must earn on its sinking fund in order to break even must now be 7.86%.

3 If the dual-rate D.C.F. method is not required, the program includes the facility of making earning rate equal to paying rate for Yield and Net Present Value methods. This reduces the generalized D.C.F. method to the single-rate D.C.F. method.

4 If the sum of the undiscounted net cash flows is zero, the single-rate D.C.F. Yield solution rate of return will be given as zero. In this particular case, paradoxically, it makes no difference to

the Yield figure whether the initial investment is recovered in 1 or 100 years. However, relative profitability can easily be established by comparing Net Present Values at a discount rate greater than zero.

5 It may be necessary to calculate repayment of capital investment in real purchasing power terms and to adjust discount rates to this. Normally the assumption is made that real purchasing power remains constant throughout the project period. If this is not so, it is possible to convert the Yield solution rate to real purchasing power terms (assuming cash flow forecasts not previously adjusted to equivalent in real purchasing power terms) as follows:

Let r be Yield solution rate output by program

r_1 be Yield solution rate in real purchasing power terms

x be annual rate of inflation (positive) or deflation (negative) of money values, constant throughout duration of project.

then $r_1 = \left(\frac{100 + r}{100 + x} - 1 \right) \times 100$

The discount rate for the Net Present Value calculation can also be converted as follows (assuming cash flow forecasts not previously adjusted to equivalent in real purchasing power terms):

Let r be cost of capital or opportunity cost

r_1 be converted discount rate to be input into program

x be as x in previous equation

then $r_1 = \frac{(100 + r)(100 + x)}{100} - 100$

6 In case there should be any doubt as to how a compounding effect on debts or sinking funds can be simulated by discounting, the following example can illustrate the process.

(a) Accumulating debt (£) at a 10% paying rate.

	Year 1	Year 2	Year 3
Cash Flows	-100	-150	+286

Capital plus interest accrued would be as follows:

		Future Value
Year 1 -100 at 10% for two years	=	-121
Year 2 -150 at 10% for one year	=	-165
Year 3 +286	=	+286
i.e. $[-100 \times (1.1)^2] + [-150 \times 1.1] + 286$	=	0.0

In discounted form the equation would be as follows:

		Present Value
Year 1 -100	=	-100
Year 2 -150 discounted at 10% for one year	=	-136.36
Year 3 +286 discounted at 10% for two years	=	+236.36
i.e. $-100 + (-150 \div 1.1) + [+286 \div (1.1)^2]$	=	0.0

(b) The sinking fund calculation is a repetition of the above, except that signs of the cash flows will be reversed, and the discount rate will be earning rate.

3389 (11.66)

3 Probability evaluation

UNCERTAINTY 3.1

Few investment decisions can be settled by a single profit calculation. Most involve varying degrees of uncertainty indicating a number of possible profit levels. In such circumstances the decision to accept or reject a project often hinges on the manager's evaluation of uncertainty. This may require the evaluation of many variables, each entailing a search for parallel situations, and then a subtle weighting and discounting process for judging their relevance. It is seldom possible to commit such mental processes to paper since the manager himself is seldom fully aware of them. The result is that the effect of long experience and/or sustained logical thought can seldom be weighed in any single decision. Furthermore it has been found that attitudes to risk are strongly influenced by basic personality characteristics. These difficulties in evaluating estimates of probability are compounded when, as often happens, a committee decides policy on the basis of estimates prepared by staff with whom many members of the committee have no direct contact.

Despite the obvious difficulty in obtaining and handling probability estimates, they are receiving increasing attention. This is because they often represent the crucial factor in a decision. In highly capitalized companies, management committees are required to make decisions tying up large amounts of the company's capital assets. Increasing competition forces them to work to a lower margin of error, and it becomes essential to have available a detailed picture of all possible consequences of a decision. It might be found that a project promising high profits on a 'most probable' basis also carries some risk of a loss on a scale that might affect the survival of the firm. In consequence a project offering small but relatively certain profits may be preferred to one offering the prospect of high profits linked with the possibility of equally high losses. Some firms follow a policy of balancing high risk projects with steadier earners in a given ratio so as to combine long term growth and short term profit. It has also been found that the discipline of thinking out forecasts and expressing them in probability terms sharpens management planning and induces forward thinking. It seems that once a manager gets used to the effort of forecasting change, he is less likely to be surprised by it.

There are three basic attitudes to uncertainty: 1) ignore it, 2) express it verbally, or 3) express it numerically. The first is common but illusory since it invariably involves arbitrary suppression of other less or equally possible forecasts. Verbal expression of uncertainty represents an advance but is still imprecise since, for example, 'most probable' may in different circumstances mean anything between, say, 30% and 95% probability. Numerical expression of uncertainty offers much more precision in communication and handling but requires careful and specific statement of the assumptions under which the estimate is made.

PROBABILITY ESTIMATES 3.2

Probability theory must not be thought of as a crystal ball explaining the future with the aid only of formulae. It is a body of techniques that, under more or less rigorous assumptions, can be used to predict the patterns of events in the future on the basis of the patterns observed in similar events in the past. There is a basic distinction to be made between objective and subjective probability estimates.

Objective probability estimates

These are based on quantifiable evidence and known assumptions, and are open to independent recalculation. It is also desirable that they be stated in such a way that the conclusions can be proved experimentally or by actual results. The traditional examples illustrating objective probability are tossing a coin or rolling dice. One states that the probability of a coin falling heads uppermost is 50% or 0.5 because, on the assumption that the coin is two sided (the chance of it falling on its edge being negligible), one heads, the other tails, there seems to be no reason to expect the coin to fall on one side more than the other in the long run over a series of repeated throws. Similarly the probability of a dice falling side 3 uppermost is 16.7% or 0.167 because, if the dice is true and six-sided, there appears no reason for one side to turn up more frequently than any other over an extended series of throws. On this basis certain other rules can be stated. The addition rule states that the probability of either of a number of values occurring is equal to the sum of their individual probabilities. Thus the probability of 3 or 4 occurring on a dice throw is equal to 0.167 + 0.167, i.e. 0.334 or 33.4%. The multiplication rule states that the probability of two values occurring in successive throws is equal to the product of their individual probabilities. Thus the probability of 4 following 3 in successive throws is 0.167 × 0.167, i.e. 0.0278 or 2.78%. If the full range of possible values is known, together with their probabilities, the rules can be combined as follows. The probability of two successive throws of a dice showing a total of 5 is equal to the probability of the first throw showing 1, 2, 3, or 4 multiplied by the probability of the second throw showing the single 'complement' to the first throw (i.e. 5- value of first throw), i.e.:

$$(0.167 + 0.167 + 0.167 + 0.167) \times 0.167 = 0.1116 \text{ or } 11.2\%$$

Subjective probability estimates

Where the data and assumptions used in formulating the estimate are not available for independent recalculation it must be considered subjective. Such conditions apply to a greater or lesser degree in most business situations since similar problems in the past are seldom fully documented, and in any case changing circumstances soon invalidate direct comparison. A further difficulty is that there is often no way of checking the accuracy of the resulting forecasts since they will invariably be for a unique occasion. For instance, if the sales forecast for next year is 90% probability of £100000 and 10% probability of £1000, it will be very difficult to check the accuracy of the forecast, whatever the actual sales figure turns out to be since, according to its 'classical' interpretation, a probability forecast is verifiable only over long series. Like the bookmaker, management will learn to take long term profit rather than the outcome of each individual project as its index of forecasting accuracy. The conclusion is that one does not abolish intuitive hunches by expressing probability in numerical form. All one can hope to do is to express them more precisely, and work towards the position where the forecasts of experts in their particular fields can be combined into a consensus of profit expectation for the project as a whole. To achieve this one must clearly state the assumption under which each set of forecasts is to be made, and request the probability figure to be supplied by the same person who supplies the cash flow forecast. Most people are sufficiently acquainted with the principles of probability evaluation from their everyday experience to do this. The advantages of having probability estimated by those best acquainted with each particular field are the same as for having the cash flows estimated by them. The next problem is to combine probability estimates for individual cash flows into a probability estimate for the net cash flow and profit figure.

COMPUTER SIMULATION BY RANDOM NUMBER SELECTION 3.3

Once alternative cash flow forecasts are considered for each factor in a project, the total of possible combinations rises rapidly and soon gets beyond manual calculation. This can be overcome by use of the computer but even so one can reach the stage with percentage expressions of probability where it becomes useful to save computer time. In this situation an accurate picture of the probability of profit can be obtained by using the Monte Carlo technique of selection by random numbers over a given sample of runs to build up a frequency distribution table of various profit outcomes.

The following simplified example, Table 1, illustrates the problem. It consists of three distinct factor cash flows independently variable under constant assumptions. Each forecast in this project consists of a single cash flow in a single year but in other projects might well consist of a series of cash flows in different years.

Capital	*Probability*	*Costs*	*Probability*	*Income*	*Probability*
£	%	£	%	£	%
4500	20	14500	17	18500	8
5000	45	15000	60	19000	10
5250	25	16000	23	21000	45
5500	10			21500	30
				22000	7

Table 1

3389 (11.67)

The total number of combinations, and thus profit rates, is 4 × 3 × 5 = 60. If each forecast is equally probable, 60 calculations will give a full picture of profit expectations. But once variable percentage probability estimates are introduced, the effect of each forecast on profit expectations is weighted by its probability. In Table 1 the most probable outcome is £5000 capital, £15000 costs, £21000 income - a profit of 20%. This is about 36 times more probable than £4500 capital, £14500 costs and £19000 income - a profit of 0%. The effect of this probability weighting on the frequency distribution of profit can be seen from a small sample - in this case, say, 200 runs (see Notes on Form 3 for comment on sample size). This means that 200 different runs would be made over the above data to select capital, cost and income cash flows at random for each run but in such a way that 45% of the runs use £5000 and 10% use £5500 for the capital figure, 23% use £16000 for costs and 7% use £22000 for income and so on. Results are printed in the form of a frequency distribution table for the various rates or in the case of Net Present Value, as a frequency distribution table in 50 variable steps calculated in the program to cover the range between highest and lowest values.

It is important to be clear about the way in which dependent variables can be incorporated in the random number selection exercise. For the purposes of the exercise all cash flow forecasts must be divided into groups, one for each factor. For instance if the sales income forecast is 100% certain, only one forecast will be required in the sales income group. If there are five possible forecasts then there will be five in the group. Since selections of forecasts within each group are made at random all forecasts in one group must be quite independent of forecasts in any other group. That is, it must be impossible for the random selection procedure to combine forecasts prepared under different assumptions. In the previous example in Table 1 it would not be possible to include cost forecasts based on a sales level of 10000 in one group and income forecasts based on a sales level of 15000 in another group, because a randomly selected run might then combine a cost forecast for one sales level with an income forecast based on another. The forecasts in Table 1 for these independently variable factors with non-clashing assumptions might be input in the following order:

Form 2 Index No. sequence		1	2	3	4	5	6	7
Group Index No.		1	1	1	1	2	2	2
Probability	%	20	45	25	10	17	60	23
Capital	£	-4500	-5000	-5250	-5500			
Revenue	£					-14500	-15000	-16000

Form 2 Index No. sequence		8	9	10	11	12
Group Index No.		3	3	3	3	3
Probability	%	8	10	45	30	7
Capital	£					
Revenue	£	18500	19000	21000	21500	22000

It is possible to include dependent variables in the simulation exercise by collecting them inside one group. For instance suppose that in Table 1 costs of £14500 and income of £18500 or £19000 are for sales of 10000 items; costs of £15000 and income of £21000 for sales of 12000; and costs of £16000 and income of £21500 or £22000 for sales of 14000. If this were so, cost and income probabilities should match so let cost probabilities be 18% for £14500, 45% for £15000 and 37% for £16000. As before, variations in capital expenditure are quite independent of sales. In this case all cost and income forecasts could be combined into group 2 and, still maintaining the principle of non-clashing assumptions between groups, the forecasts input might look as follows:

Form 2 Index No. sequence		1	2	3	4	5	6	7
Group Index No.		1	1	1	1	2	2	2
Probability	%	20	45	25	10	8	10	45
Capital	£	-4500	-5000	-5250	-5500			
Revenue	£					-14500	-14500	-15000
Revenue	£					18500	19000	21000

Form 2 Index No. sequence		8	9
Group Index No.		2	2
Probability	%	30	7
Capital	£		
Revenue	£	-16000	-16000
Revenue	£	21500	22000

Within the groups the forecasts may be in any order, and the groups among themselves may be in any order.

The frequency table output by the simulation option (Figure 12 shows a Yield example) can easily be converted to a bar chart or frequency curve graph (also shown in Figure 12).

Profitability of different projects will be judged by the range and grouping of possible outcomes, and the most probable outcome (the mode).

3389 (11.66)

4 Presentation and decision

DECISION TREES

4.1

Improved techniques for evaluating complicated projects from many angles have brought with them the problem of presenting the issues simply and clearly to management for decision. One of the most useful aids in this respect is the decision tree. This shows in diagram form the expected effect on profits of changes in those factors over which one has control, in the light of changes in uncontrollable factors. The effect of different timing patterns can also be shown in this way. An example of a decision tree is shown in Figure 6.

The short space available in this guide to describe the decision tree technique does it less than justice and users of the probability simulation option in the package are urged to read the references in the bibliography if they are not already acquainted with them.

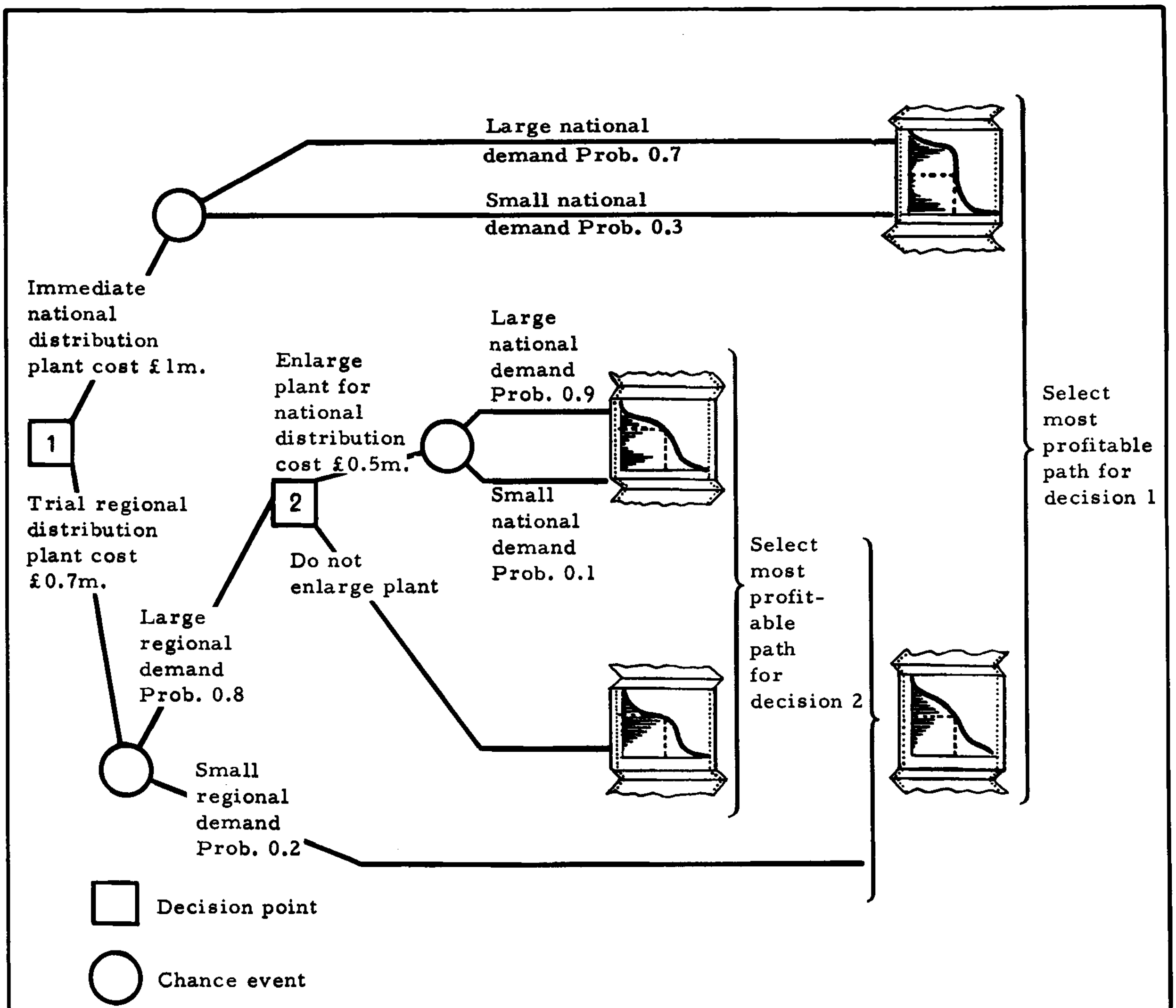


Figure 6 Decision tree for marketing brand x

OTHER FACTORS IN THE DECISION 4.2

Many projects do not lend themselves to appraisal entirely in terms of direct financial profitability. Some welfare amenities, e.g. installation of canteens are decided by reference to working conditions and their value in terms of good labour relations. Most other investment projects involve evaluation of non-quantifiable factors to greater or lesser degrees, usually by reference to 'the firm's objectives' (i.e. by answering the question. 'In five or ten years' time what kind of firm will this be, and in what kind of industry?'). Consciously or otherwise, projects on or below the borderline of acceptable profitability are judged by such criteria. This may involve putting a cash value on labour relations (welfare projects), technological leadership (research and development, new products and processes), market prestige (quality control projects), and competitive strength (diversification or vertical/ horizontal integration).

Another most important factor is the availability of sufficient skilled management and labour to make a success of new projects. Some firms have such confidence in the skills of their managements that they can enter particularly risky fields with little more than the faith that 'when the profits come, we'll have a good share in them.' In the last analysis this is often the extra factor that sways the policy of a successful firm towards expansion. Also of course, it is the executive skills of the management that in the end bring performance into line with the forecasts and justify time spent on them.

3389 (11.66)

5 1900 series PROP package

There are three different versions of the PROP (Profit Rating Of Projects) package available, each for a different 1900 machine configuration. In addition there are available a Users Guide, a brief descriptive brochure and pads of the three standard punching forms (Stationery Nos. 1/1935/1, 1/1935/2 and 1/1935/3).

MACHINE CONFIGURATION REQUIRED 5.1

All versions require the following:

Processor - - - - - - - - 1900 range with floating point facilities.

Input - - - - - - - - - - Standard 80 column punched cards or 8 track paper tape.

Output - - - - - - - - - Line printer equipped with 'Head of form' control loop and 11-inch stationery, minimum 120 print positions.

In addition the minimum core store and backing store requirements of each version are as follows:

Program Number	*Core Store*	*Backing Store*
#X4P1	16K	2 magnetic tapes, 20 kch/s minimum
#X4P2	8K	3 magnetic tapes 20 kch/s minimum
#X4P3	16K	1 EDS disc drive

OPERATING INSTRUCTIONS 5.2

PROGRAM #X4P1

1 Scratch two tapes

2 Load program tape

3 Transfer program into store by means of message FI #X4P1

4 Load Data Cards or Paper Tape

5 If paper tape data input is required it can be initiated by means of the message ON #X4P1 1

6 Activate program by means of message GO # X4P1 20

7 End of Run Halt ...... DE 11

PROGRAM #X4P2. As above with #X4P2 substituted for #X4P1

PROGRAM #X4P3

1 Load program tape

2 Transfer program into store by means of message FI #X4P3

3 Load EDS cartridge and locate by means of message GI #EXEC X Y where X is the unit number and Y the cartridge number.

4 Load Data Cards or Paper Tape

5 If data input via paper tape initiate input by means of message ON #X4P3 1

6 Activate program by means of message GO # X4P3 20

7 End of Run Halt ...... DE 11

EXCEPTION CONDITIONS 5.3

Error halts on console typewriter.

Halted 0x	Alphabetic character in numeric field. x represents the alphabetic character. Generally occurs either because of a mispunching, OR where 'Last Form 2' or 'Last Form 3' markers are missing, OR where a blank card has been omitted from the last Form 2.
Halted ET	End of tape reached. Either 'Number of Groups in Batch' (Form 3) not given, OR only one Group used in simulation. In latter case add a blank Form 2 as a second 'dummy' Group.
Halted FA Halted FB	Hardware parity error. Restart run with another disc.
Halted FC	Form 2 selected cannot be found.
Halted FF	Cartridge full - use another.
Halted I.	Decimal point in integer field.
Halted IN	Group index numbers in Forms 2 inconsistent.
Halted NC	Probability does not equal 100 in a group.

Halted PARITY ERROR - USE ANOTHER CARTRIDGE.

Halted THIS CARTRIDGE UNSUITABLE - TRY ANOTHER.

Error printouts on line printer.

ERROR 1	Either no Form 2 corresponding to selection can be found, OR selections in Form 3 not in ascending numerical order.
ERROR 2	No method specified for simulation exercise.
ERROR 3	Date of capital or revenue input outside range.

LIFE OF PROJECT ONLY 1 PERIOD. This indicates that the project Net Cash Flow is for one period only. The program abandons further calculations and moves on to the next Form 3. In the case of a simulation exercise such a result is not included in the TOTAL RUNS figure.

RESTART FACILITY 5.4

Each program includes a 'restart and salvage' facility for the simulation exercise. In the event of an irremediable tape or disc failure during the 41st or subsequent passes of a simulation exercise, the results obtained in the previous passes can be output by means of the following message(s).

Program #X4P1	GO #X4P1	*10257	
Program #X4P2	AL #X4P2	*1070	*2552
	AL #X4P2	*11664	*4
	GO #X4P2	*6451	
Program #X4P3	GO #X4P3	*11136	

6 Program input

Data is input on punched cards or paper tape from standard forms as follows:

1 FORM 1 Up to ten combinations of investment grants, initial and annual tax allowances, together with methods of tax depreciation (i.e. straight line or reducing balance) and tax lives. Corporation tax rates, frequency of time periods per annum (1 to 99), lags before tax and grants recoverable.

2 FORM 2 Up to 30 capital, scrap values and working capital investments; dates of investment and operating year together with reference to relevant tax class on Form 1; and/or up to 30 revenue cost or income amounts. Also a probability estimate if simulation option required.

3 FORM 3 Each Form 3 generates either a single run on the selected Forms 2 or a random selection simulation exercise on the whole batch. It specifies which calculations are required and the discount rates where necessary, the number of runs required in the simulation exercise, and the headings for the printouts.

Formats and notes for filling out the forms can be found in subsections 6.2, 6.3 and 6.4.

GENERAL PROGRAM ASSUMPTIONS 6.1

1 All cash flows are regarded as occurring on the last day of the appropriate period.

2 Amounts, cash grant rates, tax rates and allowances, discount percentages and accuracy limits may be input in either decimal or integer form, but must not include commas. Years must be written in consistent format, i.e. 1975, 1976, etc. *or* 1, 2, 3, 4, etc. but *not* 1968, 9, 70, etc. All figures punched must be right justified, except decimal figures.

3 There must be at least one form of each type in each batch of data input. If a form is not required, a dummy should be inserted. For details see subsections 6.2, 6.3 and 6.4.

4 The overall input limits are as follows:

Any number of batches, each batch containing:

1 Form 1
Up to 99 Forms 2
Any number of Forms 3

Each Form 3 may select up to 40 Forms 2 for inclusion in a single run. Random selection exercises cover all Forms 2 in the batch.

5 The 'year' into which data is grouped is taken to be identical to the tax year for tax calculations.

6 There is a limit to project length of 30 time periods. The time period selected may be annual, half-yearly, quarterly, monthly (calendar or four week), weekly, etc. to a limit of 99 in a year.

7 Throughout the program, signs on cash flows relate to their direction of flow into the firm from the project (+) or out of the firm to the project (-).

8 There are assumed to be enough taxable profits elsewhere in the firm to offset immediately any temporary excess of tax allowances over corporation tax payable.

9 Real purchasing power of currency remains constant throughout project. If adjustment for effect of inflation or deflation is required, it can be obtained by adjusting forecasts of cash flows and/or adjusting D.C.F. rate (see Note 5 on page 10).

10 If the integer content of a number for the single-run printout should exceed the limit allocated for it the whole number will still be printed, preceded by an asterisk (*), and the rest of the line format will be shifted to the right for the required number of print positions, overflowing into the following line if necessary.

I·C·T

1900 Series PROP
Form 1

Taxation and investment incentive rates for

Project | **Aspect of Project**
Date
Prepared by
Checked by | **Assumptions/Comments**

No. Periods Per Annum	Tax Time Lag	Grant Time Lag	Corporation Tax Rate 1	No. of Periods	Corporation Tax Rate 2	No. of Periods	Corporation Tax Rate 3
1 2	3 4	5 6	7 10	11 12	13 16	17 18	19 22

Tax Class No.	Investment Cash Grant Rate	Initial Tax Allowance Rate	Annual Tax Allowance Rate 1	Method 1	Life Years 1	Annual Tax Allowance Rate 2	Method 2	Life Years 2
1 2	3 6	7 10	11 14	15	16 17	18 21	22	23 24

FORM 1/1935/1 (10.66)

Figure 7 Form 1 format

3389 (11.66)

For column references see Format Figure 7.

Each batch must contain one Form 1. Card input consists of one Header card plus up to ten cards for individual tax classes numbered from 1 to 10, the last of which must be numbered Tax Class No. 10 irrespective of the preceding tax class number. If no tax class cards are required, the Header card must be input together with one dummy card, blank except for Tax Class No. 10.

NUMBER PERIODS PER ANNUM (Columns 1 and 2)

If annual figures 01, if quarterly 04, if calendar monthly 12 etc., limit 99. All subsequent data in the batch must be expressed in terms of the time interval selected here, with the exception of annual tax rates and life years - Form 1, and the per annum rates of discount in Form 3.

TAX TIME LAG (Columns 3 and 4)

The delay (expressed in time periods as above) after the end of each tax year (which is identical to the calendar year for the purposes of the program) before corporation tax owed to the government is actually paid.

GRANT TIME LAG (Columns 5 and 6)

The delay (in time periods as above) after the period when the investment is made before the cash grant is received.

CORPORATION TAX (Columns 7 to 10, 13 to 16, or 19 to 22)

The option is given of two changes to the initial rate during the life of the project. Number of Periods (Columns 11 and 12, 17 and 18) gives the number of periods for which the preceding tax rate is applicable. If not required, they may be left blank and the initial rate will then apply throughout the project life. If the initial rate is also left blank, no tax allowances on capital or tax payments on profits or scrap values can be made.

In calculating the following grants and allowances, the program incorporates the proposals of the White Paper on Investment Incentives (Cmnd 2874) applicable to investments made after 17th January 1966.

TAX CLASS NUMBER (Columns 1 and 2)

This relates to tax class number (Form 2). Up to ten tax classes numbered from 1 to 10. Last tax class required must be numbered 10. No blank tax class cards are permitted in Form 1.

INVESTMENT CASH GRANT RATE (Columns 3 to 6)

These may be used or left blank. The time lag before the grant is received is nominated on the Header card (Columns 5 and 6).

INITIAL TAX ALLOWANCE RATE (Columns 7 to 10)

This is calculated on the balance of the investment outstanding after the cash grant (if any) has been deducted. If an annual allowance rate is required, the initial allowance is calculated as an advanced annual allowance.

ANNUAL TAX ALLOWANCE RATE 1 (Columns 11 to 14)

Like initial tax allowance, this is calculated on balance of capital investment outstanding after the cash grant (if any) has been deducted. If the method selected is straight line depreciation, and either the rate or the life is excessive, it may happen that allowances to a total in excess of 100% of the initial balance of investment outstanding are taken. If they are, the balance of investment undepreciated will be a negative figure and a compensatory deduction will be made in the tax allowance column for the last year. The excess annual tax depreciation and the compensatory sum will cancel out arithmetically but the time lag might affect a D.C.F. calculation.

METHOD 1 (Column 15)

Blank - no annual tax calculation required.
1 - straight line annual tax depreciation
2 - reducing balance annual tax depreciation.

I·C·T

1900 Series PROP
Form 2

Cash flow forecasts for

Project
Date
Prepared by
Checked by

Aspect of Project

Assumptions/Comments

Form 2 Index No.		Last Form 2 Mk.	Probability of Forecast			Group Index No.	
1	2	3	4		6	7	8

Year	Period	Capital Amount	Operating Year	Tax Class No.	Revenue Amount
1–4	5–6	7–15	16–19	20–21	22–30

FORM 1/1935/2 (10.66)

Figure 8 Form 2 format

3389 (11.66)

LIFE YEARS 1 (Columns 16 and 17)

Numbers of years for which annual tax allowances are to be calculated. The final balancing allowance, if any, will be calculated in the last year of the life (or the 30th project period if this falls sooner). This will be the scrap year. Any scrap value must be input as a separate capital flow in Form 2 (see TAX CLASS NUMBER). For tax depreciation over 100% owing to excessive lives see ANNUAL TAX ALLOWANCE RATE 1 above.

The option is given of changing annual tax rates and/or method of depreciation in ANNUAL TAX ALLOWANCE RATE 2 (Columns 18 to 21), METHOD 2 (Column 22), and LIFE YEARS 2 (Columns 23 and 24). If not required, these should be left blank.

FORM 2 FORMAT AND NOTES 6.3

For column references see Format Figure 8

Each batch may contain up to 99 Forms 2. Card input for each form consists of one Header card plus up to 30 cards, one for each Year and Capital Amount and/or Revenue Amount entry. If less than 30 of the Year and Amount cards are required, a blank card must follow the last to indicate end of form.

FORM 2 INDEX NUMBER (Columns 1 and 2)

This distinguishes each Form 2 for the purposes of selection on Forms 3. Forms 2 must be input in ascending index number order within each batch.

LAST FORM 2 MARKER (Column 3)

This marks the last Form 2 in each batch by 1, and is left blank for all others.

PROBABILITY OF FORECAST (Columns 4 to 6)

This is a positive integer percentage expressing the likelihood of the forecast or forecasts input on the same Form 2 being achieved. 100 expresses certainty. It may be left blank in the case of single runs over specified forms, but if the simulation exercise is being carried out on the batch, it must contain a probability estimate from 1 to 100. If revenue and capital sums are included in the same form, the probability estimate applies to them both, as does the GROUP INDEX NUMBER (see below). This enables dependent variables to be included in the random selection exercise.

GROUP INDEX NUMBER (Columns 7 and 8)

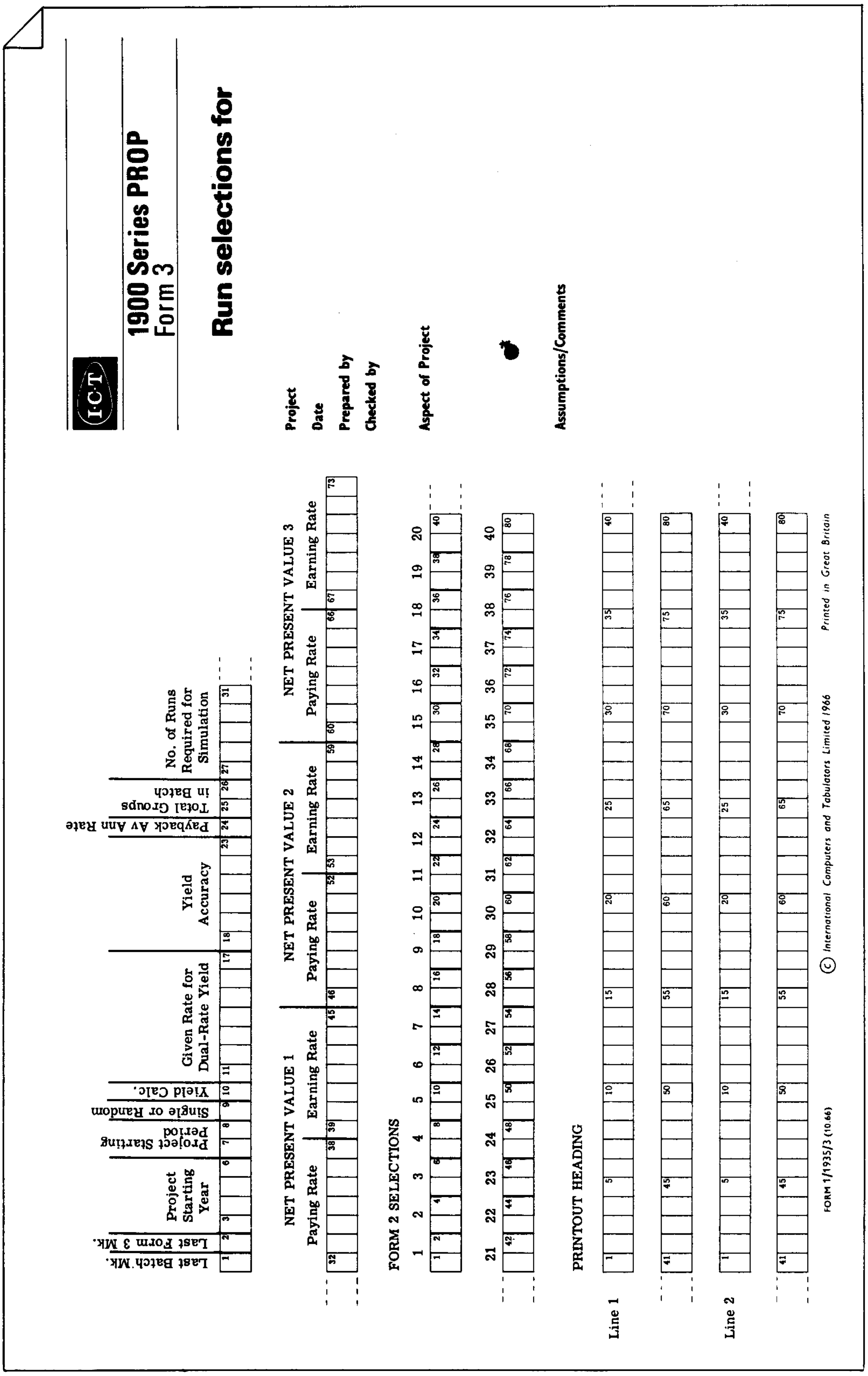
In order to carry out the simulation exercise, the Forms 2 in the batch must be divided up into groups, one for each type of cash flow, e.g. plant capital investment, working capital, running costs, sales revenue, etc. For instance, if group 1 is to be plant capital investment, all Forms 2 containing different forecasts for plant capital investment must be put in group 1. Then one form in group 1 will be selected at random for each of the simulation runs. The probability estimates for all Forms 2 in each group *must total 100*, whether the group contains one Form 2 or fifty. As mentioned in 3.3 it is possible to include dependent variables by inputting capital and revenue amounts on the same Form 2, and thus in the same group. Independently variable capital and revenue forecasts must be input on separate Forms 2 and in separate groups. Each batch, and thus each project, may contain between 2 and 99 different groups from each of which a random selection of a Form 2 will be made for each run of a simulation exercise. In each batch *either* all Forms 2 must be given group index numbers *or* all Forms 2 must have blank group index numbers.

YEAR (Columns 1 to 4)

This shows the year of input of capital and/or revenue amounts. Any order of sequence within the form is accepted. It may be either the calendar or the tax year, but the latter is necessary whenever taxation or tax allowances are calculated since they are accumulated within each year and cash values are taken after the TAX TIME LAG (see Form 1). Blank years in data are not allowed.

PERIOD (Columns 5 and 6)

Number of the period in the year in which the capital and/or revenue amount is input. Length of period must be consistent throughout the batch (see Form 1). Blank periods in data are not allowed.


I.C.T

1900 Series PROP
Form 3

Run selections for

Project
Date
Prepared by
Checked by

Aspect of Project

Assumptions/Comments

Last Batch Mk. (1)
Last Form 3 Mk. (2)
Project Starting Year (3–6)
Project Starting Period (7–8)
Single or Random (9)
Yield Calc. (10)
Given Rate for Dual-Rate Yield (11–17)
Yield Accuracy (18–23)
Payback Av Ann Rate (24)
Total Groups in Batch (25–26)
No. of Runs Required for Simulation (27–31)

NET PRESENT VALUE 1: Paying Rate (32–38), Earning Rate (39–45)
NET PRESENT VALUE 2: Paying Rate (46–52), Earning Rate (53–59)
NET PRESENT VALUE 3: Paying Rate (60–66), Earning Rate (67–73)

FORM 2 SELECTIONS
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40

PRINTOUT HEADING
Line 1
Line 2

FORM 1/1935/3 (10.66)

© International Computers and Tabulators Limited 1966

Printed in Great Britain

Figure 9 Form 3 format

CAPITAL AMOUNT (Columns 7 to 15)

Limits: 9-digit positive integer number, 8-digit negative integer number (preceded by sign), 8-digit positive decimal number (plus point), 7-digit negative decimal number (plus point and preceded by sign). See General Program Assumptions 1, 2 and 7.

OPERATING YEAR (Columns 16 to 19)

Year in which the preceding capital investment comes into operation. Annual tax depreciation may only begin in this year. Operating year is made equal to input year (Columns 1 to 4) if the former is left blank or falls earlier than the latter.

TAX CLASS NUMBER (Columns 20 and 21)

This specifies the tax class on Form 1 that is applicable to the preceding capital amount. If the tax class number on Form 2 is left blank the preceding capital amount is treated as a scrap value and corporation tax is levied upon it. If the tax class number on Form 2 specifies a card in Form 1 that is blank except for the appropriate tax class number, no tax calculations whatever are performed on the preceding capital amount and it passes unaltered into the net cash flow column. This is suitable for working capital (and any recovery of working capital must be input in the same way but with positive sign) or for the input of values that are required to be discounted or compounded without any further cash grants, tax allowances or corporation tax being taken. Negative capital amounts are grouped in the Capital Investment column of the printout. Positive capital amounts are grouped in the Other Capital column of the printout.

REVENUE AMOUNT (Columns 22 to 30)

Revenue cost and/or income amounts, negative or positive. The program prints out the negative amounts in the Revenue Costs column, the positive amounts in the Revenue Income column and adds the two together for the Profit column. Corporation tax is then calculated on the total profit for each year, its sign is reversed, and it is then entered into the Tax on Profits column after the appropriate Tax Time Lag (Form 1) or in the 30th project period if this falls sooner. Both capital and revenue amounts may be input on the same card under the same Year and Period if convenient. Revenue and capital input on the same Form 2 are considered in the same Group Index Number if one is nominated. This may be useful if they are dependently variable or 100% certain and independent, but otherwise they must be input on different Forms 2 and under different groups for the random simulation.

FORM 3 FORMAT AND NOTES

For column references see Format Figure 9

Any number of Forms 3 may be input in each batch. Each Form 3 consists of one Header card, one card for Form 2 selections, and two cards giving the heading for each sheet of the printout for that run.

LAST BATCH MARKER (Column 1)

This indicates by a 1 that there are no more batches to be read in after this. Otherwise it is blank. It is only read if the LAST FORM 3 MARKER (Column 2) shows 1.

LAST FORM 3 MARKER (Column 2)

This indicates by a 1 the last Form 3 of each batch and is left blank for all others.

PROJECT STARTING YEAR (Columns 3 to 6) and PROJECT STARTING PERIOD (Columns 7 and 8)

These contain the first year and period to be shown on the printout. This represents the present for the discounting operations in the D.C.F. method. Note that all cash flows must fall on or after this period, but not more than 29 periods after. The project starting period need not be the first period of the year.

SINGLE OR RANDOM (Column 9)

Blank - a single run
1 - a random simulation run.

The simulation exercise ignores any selections made in the following Form 2 selection card, which may be left blank. The printout heading cards however are still used.

YIELD CALCULATION REQUIRED (Column 10)

Blank - Yield calculation not required
1 - Earning rate given, find paying rate
2 - Paying rate given, find earning rate
3 - Single-rate calculation required.

GIVEN RATE FOR DUAL RATE YIELD (Columns 11 to 17)

May be integer or decimal, positive or negative. If Column 10 contains 1, given rate is earning rate, if it contains 2 then given rate is paying rate, if it contains 3 or is blank, the given rate is not required and may be left blank.

YIELD ACCURACY (Columns 18 to 23)

Since the Yield calculation 'homes' onto the solution rate, it is useful to nominate the accuracy required and thus save unnecessary calculating time. The calculation stops either when the approximate solution rate is within ± (Yield Accuracy) of the true solution, or where no solution has been found between ± 112%. The lower the accuracy required, the more time is saved, especially in the simulation exercise.

PAYBACK AND AVERAGE ANNUAL RATE OF RETURN CALCULATION (Column 24)

Blank - Neither required
1 - Payback Period calculation required
2 - Average Annual Rate of Return required
3 - Payback and Average Annual Rate of Return required.

TOTAL NUMBER OF GROUPS IN BATCH (Columns 25 and 26)

This is used as a check to ensure that each random simulation run takes one estimate from each group in the batch. If the random simulation has not been selected in Column 9 of this card, this may be left blank.

TOTAL NUMBER OF RANDOM RUNS REQUIRED (Columns 27 to 31)

This is the number of runs to be used in the random simulation exercise. A useful rough index would be the number of runs required before the least probable forecasts in each group could be expected to have combined in one run. For instance, if in a three-group project, the least probable forecasts are 5%, 25% and 40% respectively, a rough idea of the runs required to include the least possible combination would be

$$\frac{100}{5} \times \frac{100}{25} \times \frac{100}{40} = 200$$

This index rises very steeply as the number of groups increases and very soon reaches the point where one cannot hope to cover all the combinations, and the governing factor then becomes machine time available. In most cases a sample of 200 would give a good idea of probability, and single runs could be used for the least probable extremes.

NET PRESENT VALUE 1, 2 and 3 (Columns 32 to 73)

Three calculations of Net Present Value at different paying and earning rates can be made on each single run. If the single-rate calculation is required, the two rates must be made equal. Discount rates may be decimal or integer, negative or positive.

The simulation exercise may be carried out using Yield (filling in Columns 10 to 23), or Net Present Value (Columns 32 to 45), or Average Annual Rate of Return (Column 24) but not more than one of these at a time.

FORM 2 SELECTIONS (Columns 1 to 80)

Up to 40 selected Forms 2 in ascending numerical order may be included in any single run. If the simulation exercise is required, these may be left blank.

PRINTOUT HEADING (Two cards, Columns 1 to 80 each)

Entries literally transferred to printout heading; any character may be punched.

7 Program output

Each Form 3 generates a printout. There are two basic printout formats according to whether a single run is requested (Form 3, Column 9) or a simulation exercise.

SINGLE-RUN PRINTOUT AND NOTES 7.1

This is a two-page printout. The first page (see Figure 10) consists of the following:

HEADING

160 letters or digits transferred literally from the Form 3 generating the run.

YEAR and PERIOD

These start with the First Project Year and Period (Form 3, Columns 3 to 8) and list and date consecutively each period for the duration of the project (limit 30 periods). Period length is taken from Number Periods per Annum (Form 1, Columns 1 and 2). Year, period and zero entries in other columns after the period in which the last project cash flow occurs are suppressed. Project cash flows in the remaining ten columns relate to the equivalent year and period.

CAPITAL INVESTMENT

All amounts input through Form 2, Columns 7 to 15, and whose sign is negative (i.e. cash flow out of firm into project), are grouped in this column. The limits to each number printed out in this and the subsequent nine columns on this page are +99 999 999.9 and -9 999 999.9. Note that this is a printout limit only and does not affect the accuracy of amounts held during the program or the accuracy of the D.C.F. rates printed on the following page.

OTHER CAPITAL

All amounts input through Form 2, Columns 7 to 15, and whose sign is positive (i.e. cash flow into firm from project), are grouped in this column. These would include scrap values and recovery of working capital. If it is required to input figures without calculating tax, allowances etc., they must be input through Form 2, Columns 7 to 15 under tax classes with blank rates, lives, etc. and in the printout they will also be grouped according to sign in either the CAPITAL INVESTMENT or OTHER CAPITAL columns.

INVESTMENT GRANT

This represents the cash values of any investment (cash) grants nominated in the appropriate tax class (Form 1) for amounts input through Form 2, Columns 7 to 15. The cash values are taken after a delay equal to Grant Time Lag (Form 1, Columns 5 and 6) after the period of capital input. All values still outstanding in the 30th project period are taken in that period.

INITIAL ALLOWANCE

This represents the cash values of any Initial Tax Allowances nominated in the appropriate tax class (Form 1) for amounts input through Form 2, Columns 7 to 15. The cash values are taken after a delay equal to Tax Time Lag (Form 1, Columns 3 and 4) after the end of the year of capital input. All values still outstanding in the 30th project period are taken in that period.

ANNUAL ALLOWANCE

This represents the cash values of any Annual Allowances nominated in the appropriate tax class (Form 1) for amounts input through Form 2, Columns 7 to 15. The cash values are taken after a delay equal to Tax Time Lag (Form 1,Columns 3 and 4) after the end of the year in which they fall due.

PLANT EXPANSION - BRAND X. ASSUMPTIONS - NO IN/DEFLATION OF COSTS/PRICES

GRANT 20%. ANNUAL ALLOW. 25%. REDUCING BALANCE FOR 5 YEARS. CORPORATION TAX 40%

YEAR	PERIOD	CAPITAL INVESTMENT	OTHER CAPITAL	INVESTMENT GRANT	INITIAL ALLOWANCE	ANNUAL ALLOWANCE	REVENUE INCOME	REVENUE COSTS	REVENUE PROFIT	TAX ON PROFIT	NET CASH FLOW
1966	3	-10000.0	0.0	0.0	0.0	0.0	0.0	-500.0	-500.0	0.0	-10500.0
	4	-2000.0	0.0	0.0	0.0	0.0	3000.0	-1000.0	2000.0	0.0	0.0
1967	1	0.0	0.0	0.0	0.0	0.0	2500.0	-1000.0	1500.0	0.0	1500.0
	2	0.0	0.0	0.0	0.0	0.0	2000.0	-1000.0	1000.0	0.0	1000.0
	3	0.0	0.0	0.0	0.0	800.0	3000.0	-1000.0	2000.0	-600.0	2200.0
	4	0.0	0.0	0.0	0.0	0.0	2500.0	-1000.0	1500.0	0.0	1500.0
1968	1	0.0	0.0	2000.0	0.0	0.0	2500.0	-1000.0	1500.0	0.0	3500.0
	2	0.0	0.0	0.0	0.0	0.0	2000.0	-1000.0	1000.0	0.0	1000.0
	3	0.0	0.0	0.0	0.0	600.0	3000.0	-1000.0	2000.0	÷2400.0	200.0
	4	0.0	0.0	0.0	0.0	0.0	2500.0	-1500.0	1000.0	0.0	1000.0
1969	1	0.0	0.0	0.0	0.0	0.0	2000.0	-1500.0	500.0	0.0	500.0
	2	0.0	0.0	0.0	0.0	0.0	2000.0	-1500.0	500.0	0.0	500.0
	3	0.0	0.0	0.0	0.0	450.0	2500.0	-1500.0	1000.0	-2200.0	-750.0
	4	0.0	0.0	0.0	0.0	0.0	2500.0	-2000.0	500.0	0.0	500.0
1970	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3	0.0	0.0	0.0	0.0	337.5	0.0	0.0	0.0	-1000.0	-662.5
	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1971	1	0.0	800.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	800.0
	2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3	0.0	0.0	0.0	0.0	1012.5	0.0	0.0	0.0	0.0	1012.5
	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1972	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3	0.0	0.0	0.0	0.0	-320.0	0.0	0.0	0.0	0.0	-320.0

Figure 10 Page 1 of single-run printout

PLANT EXPANSION - BRAND X. ASSUMPTIONS - NO IN/DEFLATION OF COSTS/PRICES

GRANT 20%. ANNUAL ALLOW. 25%. REDUCING BALANCE FOR 5 YEARS. CORPORATION TAX 40%

D.C.F. YIELD RATE OF RETURN = 17.9277 % PAYING RATE PER ANNUM. ACCURATE TO WITHIN 0.00001. 5.0000 % EARNING RATE

D.C.F. NET PRESENT VALUE OF PROJECT = 1748. AT 7.0000 % PAYING & 5.0000 % EARNING RATES PER ANNUM

D.C.F. NET PRESENT VALUE OF PROJECT = 1806. AT 7.0000 % PAYING & 4.0000 % EARNING RATES PER ANNUM

D.C.F. NET PRESENT VALUE OF PROJECT = 1677. AT 8.0000 % PAYING & 4.0000 % EARNING RATES PER ANNUM

PAYBACK PERIOD ; 1 YEARS 3 PERIODS

AVERAGE ANNUAL RATE OF RETURN = 4.14 %

PRESENT VALUES OF CASH FLOWS

		AT 5.0000 % EARNING 17.9277 % PAYING		AT 5.0000 % EARNING 7.0000 % PAYING		AT 4.0000 % EARNING 7.0000 % PAYING		AT 4.0000 % EARNING 8.0000 % PAYING	
YEAR	PERIOD	PRESENT VALUE	CUMULATIVE PRESENT VALUE	PRESENT VALUE	CUMULATIVE PRESENT VALUE	PRESENT VALUE	CUMULATIVE PRESENT VALUE	PRESENT VALUE	CUMULATIVE PRESENT VALUE
1966	3	-10500.0	-10500.0	-10500.0	-10500.0	-10500.0	-10500.0	-10500.0	-10500.0
	4	0.0	-10500.0	0.0	-10500.0	0.0	-10500.0	0.0	-10500.0
1967	1	1381.3	-9118.7	1450.1	-9049.9	1450.1	-9049.9	1443.4	-9056.6
	2	883.7	-8235.0	950.5	-8099.4	950.5	-8099.4	943.9	-8112.7
	3	1865.6	-6369.5	2056.1	-6043.3	2056.1	-6043.3	2037.0	-6075.7
	4	1220.6	-5148.9	1378.4	-4664.9	1378.4	-4664.9	1362.4	-4713.3
1968	1	2733.0	-2415.9	3162.2	-1502.7	3162.2	-1502.7	3118.4	-1594.8
	2	749.3	-1666.5	888.3	-614.4	888.3	-614.4	874.0	-720.9
	3	143.8	-1522.7	174.7	-439.7	174.7	-439.7	171.5	-549.4
	4	690.0	-832.7	877.0	437.3	886.5	446.8	866.8	317.5
1969	1	331.1	-501.6	442.6	879.9	453.3	900.1	453.3	770.8
	2	317.7	-183.9	437.2	1317.1	448.9	1349.0	448.9	1219.6
	3	-457.3	-641.2	-647.9	669.2	-666.7	682.2	-666.7	552.9
	4	292.6	-348.7	426.7	1095.9	440.2	1122.4	440.2	993.1
1970	1	0.0	-348.7	0.0	1095.9	0.0	1122.4	0.0	993.1
	2	0.0	-348.7	0.0	1095.9	0.0	1122.4	0.0	993.1
	3	-342.5	-691.2	-545.0	550.8	-566.3	556.1	-566.3	426.7
	4	0.0	-691.2	0.0	550.8	0.0	556.1	0.0	426.7
1971	1	380.9	-310.3	642.3	1193.1	670.6	1226.6	670.6	1097.3
	2	0.0	-310.3	0.0	1193.1	0.0	1226.6	0.0	1097.3
	3	549.1	238.8	793.3	1986.5	832.2	2058.8	832.2	1929.5
	4	0.0	238.8	0.0	1986.5	0.0	2058.8	0.0	1929.5
1972	1	0.0	238.8	0.0	1986.5	0.0	2058.8	0.0	1929.5
	2	0.0	238.8	0.0	1986.5	0.0	2058.8	0.0	1929.5
	3	-238.8	-0.0	-238.8	1747.7	-252.9	1805.9	-252.9	1676.6

Figure 11 Page 2 of single-run printout

3389 (11.66)

All values still outstanding in the 30th project period are taken in that period. Also included in this column (and lagged similarly) are the cash values of corporation tax levied upon any scrap values (input through Form 2, Columns 7 to 15 but with a blank Tax Class Number).

All grant allowance and corporation tax values have an opposite sign to the capital sum they refer to. Thus annual allowances on negative capital investments are positive (unless straight line allowances in excess of 100% of the initial balance have been taken, in which case a compensatory negative adjustment is made in the last year of Life Years 1 or 2, Form 1, Columns 16 and 17 or 23 and 24) and corporation tax payments on positive scrap values are negative. If both initial and annual tax allowances are selected for a given investment, the initial allowance is treated as an advanced annual allowance.

REVENUE INCOME

All positive amounts input through Form 2, Columns 22 to 30 are grouped in this column.

REVENUE COSTS

All negative amounts input through Form 2, Columns 22 to 30 are grouped in this column.

REVENUE PROFITS

The period-by-period totals of the Revenue Income and Costs values.

TAX ON PROFIT

This represents the cash values of corporation tax payable on the Revenue Profit of the preceding column. The sign of the tax payable is the reverse of that of the Revenue Profit. Thus for a negative amount (loss) in the Revenue Profit column, a positive tax 'allowance' is calculated that can be used (it is assumed) to offset tax payments elsewhere in the firm for the same tax year. Cash values in this column are taken after a delay equal to Tax Time Lag (Form 1, Columns 3 and 4) after the end of the year in which the Revenue Profit falls. Any tax on profit outstanding in the 30th project period is taken in that period.

NET CASH FLOW

This is the sum total of Capital Investment, Other Capital, Investment Grant, Initial Allowance, Annual Allowance, Revenue Profit and Tax on Profit for each project period.

The second page (see Figure 11) of the single-run printout consists of the following:

HEADING

Same heading as for first page, transferred literally from the appropriate Form 3.

D.C.F. YIELD RATE OF RETURN

Solution rate printed out if required, limits +9999.9999% and -999.9999%. Also whether solution rate is PAYING, EARNING or SINGLE. The accuracy of the calculation (Form 3, Columns 18 to 23) is reprinted, together with the rate given, in the case of dual-rate calculation, and whether it is EARNING or PAYING rate. The limits to the search for a D.C.F. Yield solution are ±112% and -96% per *per period.* This limit does not affect the conversion of a *per period* solution rate to an *annual* rate for the detailed single-run printout. If no solution rate is found in this range, NO YIELD SOLUTION RATE FOUND WITHIN RANGE is substituted (see also 8.2).

D.C.F. NET PRESENT VALUE OF PROJECT

The value is printed, limits +999 999 999.9 and -99 999 999.9, also the paying and earning rates nominated. This is printed up to three times, once for each N.P.V. calculation nominated in Form 3.

PAYBACK PERIOD

The period before the initial investments can be paid back (ex interest) from earnings is printed in years and periods. If not required, this is not printed.

AVERAGE ANNUAL RATE OF RETURN

If required, this rate is printed.

```
 I.C.T. PROP                          SIMULATION OF NET PRESENT VALUE FOR EXPANSION OF BRAND X
                                      ASSUMPTION -  NO COST/INCOME IN/DEFLATION, 5% GROWTH IN MARKT

 YIELD SINGLE                                        NO RATE GIVEN
   SOLUTION  FREQUENCY                                                                     X  FREQUENCY AS %
                                                                                           *  CUMULATIVE % FREQUENCY
      34 & >     0 *
      33         0 *
      32         0 *
      31         0 *
      30         0 *
      29         0 *
      28         0 *
      27         0 *
      26         0 *
      25         0 *
      24         0 *
      23         0 *
      22         0 *
      21        10 IXXXXXXXXXX*
      20         0 I          *
      19         0 I          *
      18         2 IXX          *
      17         0 I            *
      16         0 I            *
      15         0 I            *
      14         0 I            *
      13         0 I            *
      12         0 I            *
      11        22 IXXXXXXXXXXXXXXXXXXXXXX            *
      10         0 I                                  *
       9        10 IXXXXXXXXXX                                  *
       8        23 IXXXXXXXXXXXXXXXXXXXXXXX                                            *
       7        16 IXXXXXXXXXXXXXXXX                                                                   *
       6         1 IX                                                                                   *
       5         4 IXXXX                                                                                    *
       4         4 IXXXX                                                                                        *
       3         0 I                                                                                            *
       2         0 I                                                                                            *
       1         2 IXX                                                                                            *
       0         0 I                                                                                              *
      -1         0 I                                                                                              *
      -2         0 I                                                                                              *
      -3         4 IXXXX                                                                                              *
      -4         0 I                                                                                                  *
      -5         2 IXX                                                                                                  *
      -6         0 I                                                                                                    *
      -7         0 I                                                                                                    *
      -8         0 I                                                                                                    *
      -9         0 I                                                                                                    *
     -10         0 I                                                                                                    *
     -11         0 I                                                                                                    *
     -12         0 I                                                                                                    *
     -13         0 I                                                                                                    *
     -14         0 I                                                                                                    *
     -15         0 I                                                                                                    *
     -16         0 I                                                                                                    *
     -17         0 I                                                                                                    *
     -18 & <     0 I                                                                                                    *

  TOTAL RUNS   100 I.........I.........I.........I.........I.........I.........I.........I.........I.........I.........I
                   0%        10%       20%       30%       40%       50%       60%       70%       80%       90%       100
```

Figure 12 Yield random simulation printout and comments

PRESENT VALUES and CUMULATIVE PRESENT VALUES

Both these are printed at Yield solution rate if specified in Form 3, and at the rates selected for the Net Present Value calculations in Form 3.

SIMULATION PRINTOUT AND NOTES 7.2

The single-page printout (see Figure 12) consists of the following:

HEADING

160 letters or digits transferred literally from the Form 3 generating the run.

YIELD PAYING/EARNING/SINGLE SOLUTION *or* NET PRESENT VALUE *or* AVERAGE ANNUAL RATE OF RETURN

Whichever of these has been selected in Form 3 for the method. Following this, FREQUENCY refers to the number of runs that returned the appropriate value. Then RATES GIVEN in the case of Net Present Value or dual-rate Yield.

Then follows a table of values selected within the program to span the most probable values expected from the simulation exercise: those above the range are grouped in the highest value and those below the range are grouped in the lowest value.

Then a Frequency column is printed showing the number of times each value occurred during the sample number of runs (nominated in Form 3, Columns 27 to 31).

Finally a check total of runs is printed together with (in the Yield case) the number of runs in which no solution was found between ±112% or, in the case of Net Present Value, ±8,388,607.

The right-hand side of the printout is used to represent results in visual form against a scale at the foot of the printout. The bar-chart shows each frequency as a percentage of the total. For instance, in Fig 12, a Yield between 8 and 9% was obtained on 23 out of 100 occasions. The bar shows this to be equal to 23%. The bar-chart has the following advantages:

1 Gives percentage equivalents of frequencies.

2 Shows range of possible outcomes at a glance.

3 Shows most probable grouping of outcome within total range.

In addition the cumulative percentage frequency curve (shown by asterisks) shows the overall probability of equalling or exceeding each value. For instance, in Fig 12, if cost of capital is 8% then by reading from 8% on the left-hand scale to the asterisk, and then down to the percentage scale, one finds the overall probability of equalling or exceeding 8% to be 67%.

The simulation printout now includes a bar chart giving individual frequencies as percentages and also a cumulative percentage frequency curve. This shows at a glance the probabilities of individual values and also the probability of achieving or exceeding any given value. The format of the simulation printout is shown in Fig. 12.

Note that the asterisk (*) showing cumulative frequency suppresses the last X of the first frequency bar.

ICT

1900 Series PROP
Form 1

Taxation and investment incentive rates for

Project BRAND X PLANT EXPANSION **Aspect of Project**

Date 10/8/66

Prepared by AB

Checked by CD

Assumptions/Comments NO DEVELOPMENT AREA ALLOWANCE

No. Periods Per Annum	Tax Time Lag	Grant Time Lag	Corporation Tax Rate 1	No. of Periods	Corporation Tax Rate 2	No. of Periods	Corporation Tax Rate 3
4	3	6	40	30			

Tax Class No.	Investment Cash Grant Rate	Initial Tax Allowance Rate	Annual Tax Allowance Rate 1	Method 1	Life Years 1	Annual Tax Allowance Rate 2	Method 2	Life Years 2
1	20		25	2	5			
10								

FORM 1/1935/1 (10.66) © International Computers and Tabulators Limited 1966 Printed in Great Britain

Figure 13 Completed example of Form 1 for printout Figures 10 and 11

I·C·T

1900 Series PROP
Form 2

Cash flow forecasts for

Project BRAND X PLANT-EXPANSION
Date 10/8/66
Prepared by AB
Checked by GD

Aspect of Project INVESTMENT CAPITAL, WORKING CAPITAL, SCRAP VALUE, RUNNING COSTS, AND INCOME.

Assumptions/Comments NO INFLATION OR DEFLATION OF COSTS OR PRICES. SALES CALCULATED ON 5% PER ANNUM MARKET GROWTH.

Form 2 Index No. (1–2)	Last Form 2 Mk. (3)	Probability of Forecast (4–6)	Group Index No. (7–8)
1	1		

Year (1–4)	Period (5–6)	Capital Amount (7–15)	Operating Year (16–19)	Tax Class No. (20–21)	Revenue Amount (22–30)
1966	3	-10000		1	-500.0
1966	4	-2000.0		10	3000
1967	1				2500
1967	2				2000
1967	3				3000
1967	4				2500
1968	1				2500
1968	2				2000
1968	3				3000
1968	4				2500
1969	1				2000
1969	2				2000
1969	3				2500
1969	4				2500
1971	1	800.			
1966	4				-1000
1967	1				-1000
1967	2				-1000
1967	3				-1000
1967	4				-1000
1968	1				-1000
1968	2				-1000
1968	3				-1000
1968	4				-1500
1969	1				-1500
1969	2				-1500
1969	3				-1500
1969	4				-2000
BLANK CARD					

FORM 1/1935/2 (10.66)

Figure 14 Completed example of Form 2 for printout Figures 10 and 11

I·C·T

1900 Series PROP
Form 3

Run selections for

Project BRAND X PLANT EXPANSION
Date 10/8/66
Prepared by AB
Checked by CD

Aspect of Project

Assumptions/Comments SINGLE RUN REQUIRED, NO ALTERNATIVE FORECASTS AVAILABLE FOR SIMULATION.

Last Batch Mk.	Last Form 3 Mk.	Project Starting Year	Project Starting Period	Single or Random	Yield Calc.	Given Rate for Dual-Rate Yield	Yield Accuracy	Payback Av Ann Rate	Total Groups in Batch	No. of Runs Required for Simulation
1	1	1966	3		1	5.	.00001	3		

NET PRESENT VALUE 1		NET PRESENT VALUE 2		NET PRESENT VALUE 3	
Paying Rate	Earning Rate	Paying Rate	Earning Rate	Paying Rate	Earning Rate
7	5	7	4	8	4

FORM 2 SELECTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																			

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

PRINTOUT HEADING

Line 1
PLANT EXPANSION - BRAND X. ASSUMPTI
ONS - NO IN/DEFLATION OF COSTS/PRICES

Line 2
GRANT 20%. ANNUAL ALLOW. 25%. REDUCING B
ALANCE FOR 5 YEARS. CORPORATION TAX 40%

FORM 1/1935/3 (10.66)

Printed in Great Britain

Figure 15 Completed example of Form 3 for printout Figures 10 and 11

8 Package applications

SIMPLE DISCOUNTING OR COMPOUNDING OPERATIONS 8.1

Series of cash values can be discounted back to present values or net present value. If a future value of a series compounded to a given date is wanted, the series can be input 'in reverse order', discounting each cash flow at an equivalent negative discount rate for the same period in the past as it should be compounded in the future. For this purpose the equivalent negative discount rate will be

$$-\left(\frac{\text{Compound Rate} \times 100}{100 + \text{Compound Rate}}\right)$$

i.e. multiplying by 1.1. is equivalent to dividing by 0.90909; multiplying by 1.25 is equivalent to dividing by 0.8, etc. The following examples illustrate this equivalence:

Present value				*Future value*	
1	2	3	4	5	Years
		£100 ———	———	——►£121	Compounding £100 for two years at 10% is equivalent to discounting £100 for two years at -9.09%.
£121◄———	———	£100			
			£100 ——►	£125	Compounding £100 for one year at 25% is equivalent to discounting £100 for one year at -20%.
£125◄——	£100				
		£100 +	£200 ►	£320.025	Compounding £100 in year 3 and £200 in year 4 to a Future Value in year 5 of £320.025 at 5% is equivalent to discounting £100 in year 3 and £200 in year 2 back to a Present Value in year 1 of £320.025 at -4.762%.
£320.025◄	£200 +	£100			

PROJECT PROFIT APPRAISAL, BUY OR LEASE COMPARISON, PURCHASE PRICE CALCULATION 8.2

Each project can be thought of as an opportunity to invest funds at one period in order to obtain a return flow of funds at another period. Since funds for capital investment are scarce and are bid for by users, it is necessary to use them in those projects that can return most profit to the user over and above the price (paying rate), which the user must pay their original owner when returning the funds. D.C.F. Yield solution rate and Net Present Value represent different ways of measuring the financial profit in a project. Both can be thought of as ways of expressing bids for projects. Their accuracy as guides to profitability are subject to the following conditions:

1 discount rates are set at realistic levels

2 capital and interest are repayable at will (see subsection 2.4, Note 2 if this is not the case)

3 inaccuracies in the cash flow forecasts may cause realized profit to be different from forecast profit (see section 3)

4 non-financial considerations (see subsection 4.2) may override selection by profit rating

5 in order to maximize overall profit on funds available one may accept less than maximum profits on individual projects, or pass over profitable projects.

D.C.F. Yield solution rate

1 Solution paying rate, earning rate given.

If the funds required for the capital investment are to be borrowed from the capital market, solution paying rate represents the *maximum* rate of interest one can afford to pay on the borrowed capital and avoid a loss. It is the break-even rate. If one can obtain capital at a lower rate the project will be profitable. The project with the higher solution paying rate offers most profit.

If the funds required for the capital investment are to be provided from the firm's internal reserves, solution paying rate represents the rate of return against which other projects competing for the funds must be judged. The funds will be most profitably employed in the project that offers the highest rate of return.

A negative solution paying rate means that capital investment cannot be recovered from net income. The project is expected to make a loss. If one has to choose between loss-making projects, the nearest-to-zero negative solution paying rate represents least loss.

2 Solution earning rate, paying rate given.

For some projects the rate of interest to be paid on borrowed capital may be known and solution earning rate may be required. This will be the rate of interest that must be earned on sinking funds set up during the life of the project in order to offset later negative cash flows and break even. In this case the solution earning rate represents the *minimum* rate that must be earned on funds accruing during the project in order to offset later negative cash flows and break even. If a higher rate than this can be earned, the project will prove profitable. The project with the lowest solution earning rate offers most profit.

A negative solution earning rate means that the surplus cash flows exceed the later negative cash flows at zero earning rate. In such a case the furthest-from-zero negative solution earning rate represents greatest profit.

3 Solution single rate.

The meaning to be attached to the solution rate for the single-rate Yield method depends on the signs of the amounts in the appropriate Cumulative Present Value column in the printout (Figure 11 and, for the meaning of signs, subsection 6.1 Note 7).

If the Cumulative Present Value amounts are all negative, the meaning of the single-rate solution is exactly the same as for dual-rate solution paying rate in 1 above, where the earning rate is not used during project life.

If the Cumulative Present Value amounts are all positive, the meaning of the single-rate solution is exactly the same as for dual-rate solution earning rate in 2 above, where the paying rate is not used during project life.

If the Cumulative Present Value amounts change sign during the project life, it means that for part of the project the single-rate solution represents paying rate on capital (i.e. while Cumulative Present Value amount signs are negative), and for the rest of the project it represents the earning rate on sinking funds (i.e. while Cumulative Present Value amount signs are positive). In these circumstances it is possible for multiple-solution Yield rates to exist and the dual-rate Yield method should be employed to ensure a unique solution.

4 If 'NO YIELD SOLUTION RATE FOUND WITHIN RANGE' appears on the printout, it means that no solution rate has been found between ±112%. This indicates:

that the project is exceptionally profitable

or that the project is exceptionally unprofitable

or that the data input into the program is incomplete

or that solution earning rate has been asked for, where solution paying rate is really required, or vice versa.

3389 (11.66)

D.C.F. Net Present Value

1 Paying rate.

If the funds required for the capital investment are to be borrowed from the capital market, paying rate will be set at the rate of interest that the firm expects to have to pay to obtain the funds.

If the funds required for the capital investment are to be provided from the firm's internal reserves, paying rate will be set at the 'opportunity cost' for those funds, i.e. the rate of interest they could earn in their next most profitable application. In this event paying rate might be closely related, or identical, to earning rate.

2 Earning rate.

Earning rate is the rate of interest that is expected to be earned on any sinking funds set up during the project to offset later negative cash flows. Earning rate is also the rate at which any surplus net income arising at the end of the project, after capital investment plus interest has been repaid, is discounted back to present value. In either capacity earning rate might be identical with the 'opportunity cost' paying rate mentioned above. If they are not equal, the most likely reason would be that the period of 'capital plus interest' repayment either does not coincide with, or differs in length from, the 'sinking fund' period or the whole project life over which final net incomes are discounted back to present values. In such a case the 'opportunity cost' of funds invested in initial capital might well differ from the 'opportunity cost' of funds accruing at the end of the project's life or the rate that can be earned on sinking funds during the period. If the two functions of earning rate have different values, an average must be taken for the given value, but this is not likely to prove of great significance for most projects.

3 AMOUNT

The Net Present Value of a project, discounted at paying and/or earning rates as above, is the present value equivalent of the series of net cash flows expected from the project. It can be thought of as representing the maximum bid price that could be made for those cash flows without incurring a loss or making the project less profitable than the next most profitable project. As such it represents the break-even point. If the project can be 'bought' for less than Net Present Value, one would make a profit and, in choosing between projects, one would select the highest positive Net Present Value as the most profitable.

If the Net Present Value is negative, i.e. a loss-maker, it can still be thought of as a bid price for the cash flows. But in this case, instead of paying for the opportunity to receive them one would have to be paid a positive amount equal to the (negative) Net Present Value in order to break even on the project. And one could do this by adding the positive payment to the net cash flow in the first project period. This would either reduce the initial negative net cash flow or set up a sinking fund which, if it accrues at *paying* rate, will exactly offset later negative cash flows and enable the project to break even. If one has to choose between projects with negative Net Present Values the nearest-to-zero negative Net Present Value represents least loss, i.e. the project that would require the smallest positive sum in the first period in order to break even. Similarly, if no income cash flows have been input, the nearest-to-zero negative Net Present Value represents the project with the lowest cost burden.

The option of calculating Net Present Value at three different pairs of rates has been included because opinions may differ on the right paying rate or the right earning rate for the project (which in most cases will be the same as those for the firm). In such circumstances individuals are given the opportunity of comparing projects at the rates which seem most realistic to them. It is important however that each individual should use the same basis of comparison for different projects. A Net Present Value of one project should not be compared with a Net Present Value of another project calculated at different paying and/or earning rates unless the circumstances of each project clearly warrant such a differential between the rates (see also penultimate paragraph in this section). Even where differential pairs of rates are thought to be justified it would be advisable also to compare Net Present Values calculated at identical pairs of rates. This will show the extent to which the differential in the rates has changed the Net Present Value, and also whether this change has had any effect on the decision to accept or reject.

Bearing these qualifications in mind the following general principles are offered as a guide to the effect which changes in discount rates will have on Net Present Value. One can safely use the simple rules outlined above to compare Net Present Values calculated at identical pairs of earning and paying rates without reference to these further comments. But they may prove useful in showing how changes in discount rates affect Net Present Value and how to interpret them. The net effect on Net Present Value will depend on the particular cash flow pattern and the relative values of paying and earning rates. Of course, if a discount rate is not used in the calculation, and this can be seen from the signs in the Cumulative Present Value column, changes in it will not affect Net Present Value.

(a) When paying rate is used to discount positive cash flows to simulate repayment of previous negative cash flows it represents the rate of interest being paid. A higher rate will reduce the positive present values and thus tend to lower Net Present Value.

When paying rate is used to discount negative cash flows it represents the 'opportunity cost' of the firm's funds i.e. the rate of interest they could earn in their most profitable alternative application. A higher rate will reduce the negative present values and thus tend to raise Net Present Value.

(b) When earning rate is used to discount negative cash flows to simulate payment from previous positive cash flows it represents the rate of interest being earned. A higher rate will reduce the negative present values and thus tend to raise Net Present Value.

When earning rate is used to discount positive cash flows it represents the 'opportunity cost' of the firm's funds. A higher rate will reduce the positive present values and thus tend to lower Net Present Value.

The 'opportunity cost' effects mentioned above can be illustrated by the simple example already quoted in subsection 2.3. It shows that receipt of £24.2 in two years' time is equivalent to receipt of £20 now (= present value) to someone who can earn 10% compound interest on his funds (= opportunity cost). This means that he would be prepared to receive £20 now instead of the cash flow and still break even. Similarly, instead of paying £24.2 in two years' time (-£24.2) he would be prepared to pay £20 now (present value -£20) in order to break even. But suppose a different use turns up for his money, returning 12%. In this case he would be prepared to receive £19.3 now instead of £24.2 in two years' time and break even, or pay £19.3 now instead of £24.2 in two years' time. So that a higher earning rate means a lower break-even point (i.e. lower positive present value), while a higher paying rate means a higher break-even point (i.e. nearer zero negative present value).

If the funds available for investment are limited it may be necessary to pass over profitable projects in order to find the combination of projects which makes most efficient use of the funds available. For instance, it is not necessarily less profitable to invest £100000 to return £110000 in one year's time (Yield solution paying rate 10%, Net Present Value at 5% paying rate £4762), than to invest £50000 to return £58000 in one year's time (Yield solution paying rate 16%, Net Present Value at 5% paying rate £5238) until one knows what use can be made of the other £50000. Similarly, it may be misleading to compare the profitability of projects over time periods with different opportunity costs. For example it is not necessarily less profitable to invest £100000 to return £113000 in two years' time during which one could earn 4% on one's funds (Yield solution paying rate 6.3%, Net Present Value at 4% paying rate £4475), than to invest £100000 to return £110000 in one year's time during which one could earn 5% on ones funds (Yield solution paying rate 10%, Net Present Value at 5% paying rate £4762) until one knows what rate could be earned on the £100000 (or £110000 including interest) in the second year.

In circumstances where a choice has to be made between two projects it will often prove useful to subtract the cash flows of the one from the other, period by period, and then calculate the D.C.F. Yield solution rate or Net Present Value for the difference. This shows the 'marginal' profitability of the one over the other. In circumstances where one of the two projects must be selected, e.g. buy or lease decision, this approach will show whether the extra initial investment involved in, say, a purchase is worthwhile. 'Marginal' profitability should not be confused with 'overall' profitability however, and if the option exists of discarding both projects if they prove unprofitable the 'marginal' approach by itself will not adequately measure acceptability.

'AUDIT' REAPPRAISALS OF PROJECT PROFITABILITY 8.3

One of the most useful applications of the package will no doubt be the regular comparison during the life of a project of profit achieved with profit forecast. At the same time, any changes can be quantified, the factors most responsible can be isolated, and forecasts outstanding, together with overall expectation of profit, can be revised.

3389 (11.66)

9 Mathematical basis of the calculations

Let first net cash flow be C and all subsequent net cash flows be $F_1 \ldots F_{29}$.

PAYBACK PERIOD 9.1

Payback Period = n periods, expressed as years and periods, where

$$C + \sum_{t=1}^{t=n} F_t \geq 0$$

If $C \geq 0$, n is shown as zero irrespective of subsequent F values.

AVERAGE ANNUAL RATE OF RETURN 9.2

Average Annual Rate of Return = A, given that negated capital investment amounts listed in the Capital Investment column of the printout be $G_1 \ldots G_{30}$, as follows:

$$A = \left[\left(C+\sum_{t=1}^{t=29} F_t\right) \div \sum_{t=1}^{t=30} G_t\right] \times \frac{100}{\textit{life of project in years}}$$

D.C.F. NET PRESENT VALUE 9.3

Given annual rate r_1 converted to periodic discounting rate r as follows [x = No. of Periods per Annum (Form 1)]:

$$r = \left(\sqrt[x]{1+\frac{r_1}{100}} - 1\right) \times \frac{100}{1}$$

Single-rate Net Present Value equals S at a given annual discount percentage r_1 where

$$S = C + \sum_{t=1}^{t=29} \frac{F_t}{(1+\frac{r}{100})^t}$$

Dual-rate Net Present Value equals S in the following equation, at a paying rate (p) on funds invested and an earning rate (e) on surplus funds accruing to the firm from the project (both p and e being periodic rates equivalent to annual discount rates given). Given m_1, m_2, etc. as the point(s) t at which (C + cumulative total of discounted cash flows) changes from negative to positive, either directly or after intervening periods of zero.

Given n_1, n_2, etc. as the point(s) t at which (C+ cumulative total of discounted cash flows) changes from positive to negative, either directly or after intervening periods of zero.

Then, if C is negative,

$$S = \left\{\left[\left(C+\sum_{t=1}^{t=m_1} \frac{F_t}{(1+\frac{P}{100})^t}\right) \frac{(1+\frac{P}{100})^{m_1}}{(1+\frac{e}{100})^{m_1}} + \sum_{t=m_1+1}^{t=n_1} \frac{F_t}{(1+\frac{e}{100})^t}\right] \frac{(1+\frac{e}{100})^{n_1}}{(1+\frac{P}{100})^{n_1}} + \right.$$

$$\left. \sum_{t=n_1+1}^{t=m_2} \frac{F_t}{(1+\frac{P}{100})^t}\right\} \frac{(1+\frac{P}{100})^{m_2}}{(1+\frac{e}{100})^{m_2}} + \text{--- etc. for } t = 1 \text{ ---- } 29$$

If C is positive or zero, the above equation must be amended, e replacing p, n replacing m and vice versa.

Where $p = e$, the dual-rate equation reduces to the single-rate equation.

D.C.F. YIELD

The single-rate Yield solution rate of return equals r where, within stated accuracy limits,

$$C + \sum_{t=1}^{t=29} \frac{F_t}{(1+\frac{r}{100})^t} \approx 0$$

The dual-rate D.C.F. solution rate of return equals p where e is given, or e where p is given in the equation given above for dual-rate D.C.F. Net Present Value where $S \approx 0$, within stated accuracy limits.

Where $p = e$, the dual-rate equation reduces to the single-rate equation.

The Yield solution rate r for periodic discounting is converted to the annual solution rate r_1 shown in the printout, as follows [x = No. of Periods per Annum. (Form 1)]:

$$r_1 = \left[(1+\frac{r}{100})^x - 1\right] \times \frac{100}{1}$$

10 Bibliography

BOOKS

MERRETT A.J.; SYKES A. *The Finance and Analysis of Capital Projects* (London, Longmans, 1963) 544 pp.

The most comprehensive British treatment of the subject.

ALFRED A.M.; EVANS J.B. *Discounted Cash Flow - principles and some short cut techniques* (London, Chapman and Hall, 1965) 71 pp.

A useful introduction to the practical application of discounted cash flow. The second half of the book deals with short cut techniques for calculation by hand.

PAMPHLETS

BRITISH INSTITUTE OF MANAGEMENT *Capital Investment Projects - Methods of Evaluation* B.I.M. Information Note 42 (London, 1965) 18 pp.

Available to B.I.M. collective subscribers only.

Investment Appraisal - National Economic Development Council (H.M.S.O., London, 1965) 14 pp.

A brief survey of problems and techniques.

White Paper on Investment Incentives (*Cmnd 2874*) (H.M.S.O., London, 1966) 19 pp.

Announces rates of investment (cash) grants, initial and annual allowances applicable in development areas and elsewhere in Britain on capital invested after 17 January 1966.

ARTICLES

On dual rate D.C.F. techniques

DUGUID A.M.; LASKI J.G. The Final Attractiveness of a Project : A method of assessing it.

Operational Research Quarterly 1964 Vol. 15, No. 4.

TEICHROEW D. ROBICHECK A.A.; MONTALBANO M. An Analysis of Criteria for Investment and Financing Decisions under Certainty.

Management Science 1965 Vol. 12, No. 3.

On probability evaluation

HERTZ D.B. Risk Analysis in Capital Investment.

Harvard Business Review 1964 Vol. 42, No. 1.

HESS S.W.; QUIGLEY H.A. Analysis of Risk in Investments using Monte Carlo Techniques.
Chemical Engineering Symposium Series 42: Statistics and Numerical Methods in Chemical Engineering
American Institute of Chemical Engineering 1963.

On decision trees

MAGEE J.F. Decision Trees for Decision Making.
Harvard Business Review 1964 Vol. 42, No. 4.

MAGEE J.F. How to use Decision Trees in Capital Investment.
Harvard Business Review 1964 Vol. 42, No. 5.

HESPOS R.F.; STRASSMAN P.A. Stochastic Decision Trees for the Analysis of Investment Decisions.
Management Science 1965 Vol. 11, No. 10.

3389 (11.66)

AMENDMENT RECORD

List Number	*Item Numbers*
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to
	to

Should any additions or amendments to this Manual be necessary, they will be issued in the form of Amendment Lists.
When such changes have been incorporated, an appropriate entry should be made in the table on the left and the pink instruction sheet(s) should be filed behind this page.